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Age, Wage and Suicide:
Economic Modelling of Suicide

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Abstract

This paper formalises an individual's decision about suicide within a framework of lifetime-utility-maximisation models. This is in line with the literature on economic modelling of suicide. The novelty of the paper is to take into account income uncertainty. Income uncertainty reduces the expected utility of a risk-averse individual, making her/him more likely to commit suicide. On the other hand, income uncertainty creates a value to postponing suicide even when her/his income gets sufficiently low. This is because income uncertainty means that if things go well, she/he will get higher income in future. Thus, income uncertainty has two opposite effects on suicidal behaviour. The main objective of this paper is to construct an economic model of suicide for investigating net impacts of income uncertainty on suicidal behaviour. For this purpose, it is assumed that the wage evolves according to a stochastic process. Then, the threshold wage, below which an individual commits suicide, is derived as a function of the parameters of the stochastic process assumed for the wage evolution. Impacts of changes in these parameters on the threshold wage are calculated. With the result, the paper shows how income uncertainty affects suicidal behaviour.

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I Introduction

Suicide is a major public health problem in Australia. While youth suicide tends to draw much attention, disaggregated data by gender and age reveal that suicide is not primarily an issue among young Australians. The left graph of Figure 1 illustrates that male suicides consistently outnumbered female ones by a ratio of four to one. The right graph shows that males aged 25 to 34 and those aged 35 to 44 had the highest and the second highest age-specific suicide rates in 2004 among all age groups, respectively. More specifically, men aged 25 to 44 made up 36.9% of the total deaths from suicide in 2004. As such, middle-aged males are at high risk of suicide in Australia. This paper bears the middle-aged male suicide in mind.

What prompts middle-aged men to commit suicide? It is generally agreed that the unemployment rate among suicides is higher than for the population. The empirical work on economics of suicide also finds that male suicide rates are more responsive to changes in macroeconomic conditions than female ones in a variety of countries. These findings from aggregate data imply that economic difficulties are important elements for understanding middle-aged male suicides. As middle-aged men traditionally see themselves as breadwinners in western countries, they may mentally bear full brunt of fluctuations in earnings. The focus of this paper is those who take their own lives because of income uncertainty.

This paper formalises an individual's suicidal decision within a framework of utility-maximisation models. Section II begins with the literature survey on economics of suicide. Then, it outlines the landmark model laid out by Hamermesh and Soss (1974). In their model, an individual commits suicide when the present value of the lifetime utility depending on permanent income falls below some certain threshold. The model succinctly presents the relationship between income and suicide. As will be shown shortly, however, the model fails to take into account the possibility that income uncertainty affects suicidal decision. With the assumption of the wage following a stochastic process, Section III introduces income uncertainty into the model and extends it. From the extended model, the threshold wage, below which an individual commits suicide, is derived as a function of the parameters of the stochastic process assumed for the wage evolution. These parameters are the expected value of the wage-growth rate, its variance, and the probability of a downward jump in the wage. Section IV calculates impacts of changes in those parameters on the threshold wage, thereby showing how income uncertainty affects suicidal decision. Section V concludes.

II Literature Review

There are two economic approaches to the study of suicide, namely, macroeconomic approach and microeconomic one. In the empirical literature, suicide is often studied from a macroeconomic point of view. With aggregate data, for instance, Brainerd (2001), Huang (1996), and Yang (1992) show that male suicide rates are more responsive to changes in macroeconomic conditions than female ones in a variety of countries¹. Morrell *et al.* (1993) also examine the Australian data for the period of 1907 to 1990, thereby confirming that suicide rates for males fluctuate sharply with the peaks coinciding in timing with high unemployment. Thus, the empirical work on suicide from a macroeconomic point of view generally finds a positive correlation between the unemployment rate and the suicide rate.

What does such a link between employment status and suicide risk at an aggregate level imply for the theoretical work of suicide from a microeconomic point of view? In South Australia, for instance, Hassan (1995) finds that only 29 per cent out of 162 suicides committed in 1982 were in current employment. It appears that unemployment causes suicide. Barraclough and Hughes (1987), however, conducts case studies of nine unemployed suicides in England, thereby finding that all of them lost their jobs because of alcoholism or mental illness. This means that unemployment and suicide may arise from similar causes. Thus, it is debatable whether unemployment and other economic difficulties cause suicide.

Nonetheless, the theoretical work on economics of suicide often assumes that economic difficulties are risk factors for suicide. In the theoretical literature, suicide is typically studied from a microeconomic point of view. Under the assumption that an individual commits suicide as a consequence of rational choice, suicidal behaviour is formalised within a framework of standard microeconomic models. As is mentioned in the last section, this paper aims to add to the literature by taking into account impacts of income uncertainty on suicidal behaviour².

When modelling suicidal behaviour, attempted suicides are distinguished from completed ones. Yaniv (2001) focuses on a suicide attempter contemplating whether or not to apply for last-minute help. A mental health practitioner is assumed to contemplate whether or not to hospitalise an attempter instead of offering her/him therapy. Thus, the attempter and the practitioner face the risk of involuntary hospitalisation and that of suicide, respectively. Their interaction is formalised within a framework of game-theoretic models. Rosenthal (1993) also develops a game-theoretic model, in which a suicide attempt is a signal to evoke sympathy. The signalling-game model is composed of two players, namely, a sender and a receiver. The former may be de-

pressed or normal, but the latter does not know its type. While a normal sender needs sympathy, a receiver prefers responding sympathetically only to a depressed sender. Based on the signal strength, the receiver decides how to respond to the sender. A stronger signal is more credible, but less likely to result in the sender's survival. The normal sender deliberately selects the signal strength to evoke sympathy. Marcotte (2003), however, stresses the possibility that a suicide attempt evokes more than sympathy. If an attempt draws out intensive attention from others, it will improve the quality of the attempter's life by aiding her/him in dealing with difficulties in life. In such a case, an individual has an incentive to attempt at suicide. This idea is formalised within a framework of lifetime-utility-maximisation models. As those models suggest, a suicide attempt does not necessarily aim at death. For this reason, the present paper does not focus on attempted suicides but only on completed ones.

Completed suicides are also analysed from a microeconomic point of view. Yang (1987) applies a cost-benefit analysis to the study of suicide. The model is analogous to a demand-supply model. In the demand-side analysis, the expected benefit of suicide is the relief of distress. As a large amount of distress causes a high probability of committing suicide, an individual's demand curve for suicide is characterised as a positive relationship between the expected benefit and the probability of suicide. In the supply-side analysis, the expected cost of suicide includes the fear of death, the loss of future income, and the loss of any enjoyment that would be experienced during the rest of the life. As a high cost of suicide causes a low probability of committing suicide, an individual's supply curve of suicide is characterised as a negative relationship between the expected cost and the probability of suicide. In this static model, the intersection of the demand and supply curves determines an equilibrium probability that the individual commits suicide³.

Huang (1997) develops another microeconomic model of suicide within a framework of utility-maximisation models. Its foundation is the analogy of an individual's decision to commit suicide with a worker's decision to leave the labour market. While a worker has to decide how much time to put into work, an individual has to decide how much effort to put into living. If a worker decides to put zero hours into work, she/he will leave the labour market. Similarly, if an individual decides to put zero effort into life, she/he will leave the life market. In other words, the decision to commit suicide is characterised as a corner solution in this model.

In contrast to the two models mentioned above, the model developed by Hamermesh and Soss (1974) is dynamic, and hence can address issues of timing. They formulate the theory of suicide within a framework of lifetime-utility-maximisation

models. In the model, an individual's utility u depends upon consumption c which in turn is a function of permanent income YP . Let Z denote the present value of the individual's lifetime utility at age t . Then, it is expressed as

$$Z(t, YP) = \int_t^\omega u[c_s(YP)]e^{-\rho(s-t)} ds, \quad (1)$$

where ω and ρ are the highest attainable age and the subjective discount rate, respectively⁴. The essence of the model is that the individual commits suicide when the present value of her/his lifetime utility falls to a certain value. The model succinctly presents a relationship between income and suicide.

The Hamermesh and Soss model is a workhorse in the economic literature on suicide. Their model underlies most empirical work on economics of suicide. Recent examples include Molina and Duarte (2006), Neumayer (2003), and Viren (2005). The model is also extended in a variety of directions in theoretical work. Crouch (1979) amends the model by assuming that an individual's utility depends upon her/his own income as well as income of those others whom she/he loves. Its prediction is that the probability of an individual committing suicide increases if a person whom she/he loves dies. Koo and Cox (2008) replace permanent income in the model with relative one, which is measured as the distance to the social mean income⁵. Relative income is assumed to be an increasing function of human capital. Relative income of an unemployed person is also assumed to depreciate because the unemployed loses an opportunity to maintain human capital through on-the-job training. The modified model shows how a high unemployment rate increases a suicide rate. As is mentioned above, Marcotte (2003) further applies the model to the study of attempted suicides. Thus, the Hamermesh and Soss model is a building block in the literature on economics of suicide.

There is still a room for amendment to the Hamermesh and Soss model. As equation (1) shows, their model is founded on the permanent income hypothesis. It is the expected value of lifetime income, to which the model gives a crucial role in suicidal decision. On the other hand, the model gives no role to the variance of lifetime income. As the variance represents income uncertainty, the model fails to take into account the possibility that income uncertainty affects suicidal decision. Unless an individual is risk neutral, her/his utility is affected by income uncertainty. Therefore, the model should be amended. This paper introduces income uncertainty into the model, thereby extending it.

To take into account income uncertainty, this paper follows suggestions made by Dixit and Pindyck (1994). When formulating the theory of investment under uncertainty, they stress an analogy of the opportunity to make an irreversible investment

to an American option in finance. They argue that both have three characteristics in common. First, future returns are uncertain. Second, the action is irreversible. Third, there is flexibility in when to take the action. Due to the similarity, they suggest that the investor’s decision should be analysed within a framework of option-pricing models. This is known as the “real options” approach. Note that suicide also has the three characteristics. First, future flows of income to an individual are uncertain. Second, committing suicide is an absolutely irreversible action. Third, there is flexibility in when to commit suicide. Thus, the real options approach is available to modelling of suicidal decision.

III The Model

(i) Introducing Income Uncertainty

The real options approach, in general, begins with approximating the time evolution of the variable of interest with some stochastic process. A geometric Brownian motion is a continuous-time stochastic process that is commonly assumed in the economic literature. This stochastic process is assumed in a variety of contexts ranging from financial engineering to environmental economics and urban economics⁶. To take into account income uncertainty, this section approximates the evolution of the wage with a stochastic process that belongs to the class of geometric Brownian motions.

Let w denote the wage. For the wage to follow a simple geometric Brownian motion, its evolution is written as

$$dw = \mu w dt + \sigma w dz, \text{ where } dz = \epsilon_t \sqrt{dt}, \text{ and } \epsilon_t \sim \text{i.i.d.} N(0, 1). \quad (2)$$

The parameters μ and σ are constants, and dz is the increment of a Wiener process. The growth rate of w has the normal distribution: $dw/w \sim \text{i.i.d.} N(\mu dt, \sigma^2 dt)$. This distribution implies that the mean squared error of forecasting the future value of w grows linearly with the forecast horizon.

The approximation of the wage evolution, however, requires amendment to a geometric Brownian motion expressed in equation (2). There is a small probability that an individual loses her/his job over a short time interval. In other words, the wage can sharply and suddenly jump downwards at unknown future time. Taking into account infrequency of such jumps, the number of their arrival times may be assumed to follow a Poisson distribution. Over a short interval dt of time, the probability of a downward jump in the wage is given by λdt where λ is the mean arrival rate. Assume for simplicity that the wage drops to zero when a jump occurs. Then, the stochastic

process for the wage evolution is written as the combination of a geometric Brownian motion and a Poisson jump process:

$$dw = \mu w dt + \sigma w dz - w dq, \quad (3)$$

where

$$dq = \begin{cases} 0 & \text{with probability } 1 - \lambda dt, \\ 1 & \text{with probability } \lambda dt. \end{cases}$$

With the stochastic process (3), this paper approximates the wage evolution to introduce income uncertainty into the model.

(ii) Relationship between Wage and Income

Under the assumption of the wage following the combined Brownian motion and Jump process, the expected value of the wage at future time s is given by $E[w_s] = w_0 e^{(\mu-\lambda)s}$, where w_0 is an initial value. This implies that the expected wage at future time s conditional upon the present wage w_t is given by

$$E[w_s | w_t] = w_t e^{(\mu-\lambda)(s-t)}, \text{ for } s > t. \quad (4)$$

Let LY denote the present value of the total income of the remaining life. With equation (4), the expected value of LY is expressed as

$$\begin{aligned} E[LY_t | w_t] &= \int_t^\infty E[w_s | w_t] e^{-\rho(s-t)} ds \\ &= \int_t^\infty w_t e^{-\{\rho - (\mu - \lambda)\}(s-t)} ds \\ &= E[LY_t | w_t] = \frac{w_t}{\rho - (\mu - \lambda)}, \end{aligned} \quad (5)$$

where the subjective discount rate ρ is assumed to be greater than $(\mu - \lambda)$.

In equation (5), the infinite-horizon assumption replaces the finite-horizon one in the Hamermesh and Soss model discussed in the last section. This replacement is made primarily because the infinite-horizon assumption simplifies calculation for solving the model. The infinite-horizon assumption might appear unrealistic. This paper, however, focuses on middle-aged persons who are too young to take their own lives. In addition, this paper does not focus on the relationship between age and suicide, but on the one between income and suicide. For these reasons, the infinite-horizon assumption may be justified.

(iii) Optimisation Problem

The next step is to complete the model. For the moment, ignore the possibility that an individual commits suicide. Without an option to commit suicide, the individual

has to find the solution to the following problem at time t based on the present value of the expected total income of the remaining life:

$$\max \int_t^\infty u(c_s(\mathbb{E}[LY_t|w_t]))e^{-\rho(s-t)},$$

where the expected present value of the lifetime income at age t replaces permanent income assumed in the Hamermesh and Soss model.

Let V denote the maximised present value of an individual's lifetime utility at age t . Then, it is expressed as

$$V = \max \int_t^\infty u(c_s(\mathbb{E}[LY_t|w_t]))e^{-\rho(s-t)}. \quad (6)$$

This is called the “value function” in the language of the dynamic programming. This is also the indirect utility function in the sense that the maximum level of utility is attained for a given level of income⁷. Equations (5) and (6) clearly show that V depends upon the present wage w_t . By applying the basic technique of dynamic programming to this problem, the Bellman equation for any t becomes

$$V(w) = v(w)dt + \frac{1}{1 + \rho dt} \mathbb{E}[V(w + dw)], \quad (7)$$

where v represents an instantaneous utility⁸.

How should the Bellman equation (7) be modified when an individual has an option to commit suicide? Assume that the individual commits suicide when the present value of her/his lifetime utility falls below some certain threshold. This assumption is same as that of the Hamermesh and Soss model discussed in the last section. Let Ω denote the threshold level of utility, below which committing suicide is optimal. Then, the Bellman equation becomes

$$V(w) = \max \left\{ \Omega, v(w)dt + \frac{1}{1 + \rho dt} \mathbb{E}[V(w + dw)] \right\}. \quad (8)$$

If the present value of the lifetime utility reaches the threshold, the individual will commit suicide, thereby getting Ω immediately.

For some range of values of w , the individual achieves the maximum on the right-hand-side of equation (8) by terminating her/his life. For the other range, the individual achieves it by continuing her/his life. If V and v are monotonically increasing in w , there will be a single cut-off wage with termination optimal on one side and continuation on the other. It is this threshold wage, which the paper seeks to find from the Bellman equation (8).

IV Results

Let w^* denote the threshold wage, below which it is optimal for an individual to commit suicide. To derive w^* , the functional form of v in the Bellman equation (8) is specified as a log function⁹. As is detailed in Appendix, w^* is derived as a function of the parameters of the stochastic process (3) assumed for the wage evolution:

$$w^* = \exp \left[(\rho + \lambda)\Omega + \frac{1}{B_1} + \frac{1}{\rho + \lambda} \left(\frac{1}{2}\sigma^2 - \mu \right) \right], \quad (9)$$

where

$$B_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2(\rho + \lambda)}{\sigma^2}}. \quad (10)$$

Clearly, w^* is highly non-linear in the parameters. Among the five parameters, μ , σ , and λ are of our interest. With equations (9) and (10), impacts of changes in each of the three parameters on w^* are calculated.

Before calculation, the benchmark values of the parameters should be determined. With the data on male earnings for the sample period of 1997:Q1 to 2006:Q4, the average of the wage growth rate and its standard deviation are calculated as 0.01 and 0.006, respectively¹⁰. These numbers are set as the benchmark values of μ and σ , respectively. For the same sample periods, the rate of unemployed males looking for fulltime work is 0.063 on average. Given this number, assume for simplicity that 63 males per 1,000 unintentionally lose jobs only once. Assume also that a male wishes to work for 40 years, that is, 160 quarters in total. Then, the probability that a male loses his job in each quarter of a year is calculated as 0.0004. This number is used as the benchmark value of λ . The benchmark values for ρ and Ω are set ad hoc to 0.01 and 1, respectively. Table 1 summarises those benchmark vales of the parameters.

Figure 2 illustrates the threshold wage w^* as a function of the expected rate of the wage growth, μ . The range of values of μ is set from 0.001 to 0.02 with increments of 0.001. The other parameters are set to the benchmark values summarised in Table 1. The figure shows a negative relationship between μ and w^* . This means that people are less likely to commit suicide when they expect higher growth rates of their wages. In an economic boom, for instance, a higher rate of the wage growth can be expected. Therefore, the relationship between the two parameters leads to the implication that suicide rates tend to fluctuates with business cycles, which is consistent with that of the Hamermesh and Soss model discussed in Section II¹¹. The policy implication is that if the government fails to stabilise economic fluctuations, it will fail to stabilise suicide cycles.

Figure 3 leads to the main finding of this paper. It illustrates the threshold wage w^* as a function of the standard deviation of the wage-growth rate, σ . The range of

values of σ is set from 0.001 to 0.02 with increments of 0.001. The other parameters are set to the benchmark values. The figure shows a positive relationship between w^* and σ . As a higher value of σ represents greater income uncertainty, the relationship between the two parameters means that people are more likely to commit suicide when income uncertainty increases. This is the main finding of the paper. The finding has an important policy implication, namely, the effectiveness of the wage stabilisation for suicide prevention. Income uncertainty should be mitigated in order to reduce suicides. In this context, unemployment compensation, which is designed to mitigate the impact of unemployment on the unemployed, can mitigate the impact of income uncertainty on suicide. Formalisation of the relationship between income uncertainty and suicidal decision is a major contribution of this paper to the economic literature on suicide.

A comment on Figure 3 would be required. One might think that the finding derived from Figure 3 is inconsistent with the argument of Dixit and Pindyck (1994). They argue that uncertainty creates a value to waiting. Their argument implies that people are less likely to commit suicide when income uncertainty increases. In one sense, this is true. Income uncertainty means that if things go well, people will get higher income. Under such circumstances, an individual has an incentive to postpone committing suicide even when the wage becomes substantially low. Note, however, that this paper extends the Dixit and Pindyck model within a framework of utility-maximisation models. In so doing, the paper assumes that an individual is risk averse. Figure 3 shows that the value to waiting is totally offset by the risk aversion.

Figure 4 illustrates the threshold wage w^* as a function of the probability of a downward jump in the wage, λ . The range of values of λ is set from 0.0001 to 0.002 with increments of 0.0001. The other parameters are set to the benchmark values. The figure shows a positive relationship between w^* and λ . It is assumed that the wage jumps downwards when an individual loses her/his job. Therefore, the positive relationship between the two parameters means that people are more likely to commit suicide when they face higher probabilities of losing jobs. The relationship between w^* and λ has the same implications as the one between w^* and μ .

V Conclusion

This paper formalises suicidal behaviour within a framework of lifetime-utility-maximisation models. In line with the literature, an individual is supposed to commit suicide when the discounted lifetime utility drops to some certain threshold. The novelty of the pa-

per is to introduce income uncertainty into the model, assuming that the wage evolves according to a stochastic process. The model is analytically solved for the threshold wage, below which an individual commits suicide. The threshold wage is expressed as a function of the parameters of the stochastic process assumed for the wage evolution. They are the expected rate of the wage growth, the standard deviation of the wage-growth rate, and the probability of a downward jump of the wage. Impacts of changes in the parameters on the threshold wage are calculated to show how income uncertainty affects suicidal decision.

With aggregate data, this paper roughly estimates the parameters of the stochastic process for the wage evolution, thereby deriving the findings as follows. First, the threshold wage is negatively associated with the expected rate of the wage growth. This implies a link between suicide risk and macroeconomic conditions. Second, the threshold wage is positively associated with the standard deviation of the wage growth. As the standard deviation represents income uncertainty, the finding means that people are more likely to commit suicide as income uncertainty increases. This result leads to a policy implication that income uncertainty should be mitigated in order to reduce suicides. The result also implies that unemployment compensation designed to mitigate the impacts of unemployment on the unemployed can mitigate the impact of income uncertainty on suicide. Third, the threshold wage is positively associated with the probability of a downward jump of the wage. This result again implies a link between suicide risk and macroeconomic conditions. In summary, the model of this paper implies fluctuations of suicide rates that may be mitigated by public policy aimed at stabilising wages. Rigorous policy analysis, however, requires estimating the parameters of the stochastic process for the wage evolution in a formal way preferably with micro data.

Appendix

To find the threshold wage w^* from the Bellman equation (8), assume for the moment that the present wage w is sufficiently higher than w^* . Then, the Bellman equation (8) is simplified as equation (7). Multiplying $(1 + \rho dt)$, equation (7) becomes

$$(1 + \rho dt)V(w) = (1 + \rho dt)v(w)dt + E[V(w + dw)].$$

Let dt go to zero and rearrange the equation. Then, the Bellman equation becomes

$$\rho V(w)dt = v(w)dt + E[dV], \tag{11}$$

where $dV = V(w + dw) - V(w)$. As the value function V depends upon w containing the Wiener increment, it is necessary to use the Ito's Lemma for calculating the second term on the right-hand-side of equation (11):

$$E[dV] = \frac{1}{2}\sigma^2 w^2 V''(w)dt + \mu w V'(w)dt + \lambda[V(0) - V(w)]dt. \quad (12)$$

Remember that the value function V is also interpreted as the indirect utility function. On the other hand, equation (5) shows that the expected present value of lifetime income is zero when w is zero. Therefore, it is reasonable to assume

$$V(0) = 0. \quad (13)$$

Equation (13) is used as a boundary condition. With this boundary condition, equation (12) becomes

$$E[dV] = \frac{1}{2}\sigma^2 w^2 V''(w)dt + \mu w V'(w)dt - \lambda V(w)dt. \quad (14)$$

Substituting equation (14) into equation (11), the Bellman equation becomes

$$\frac{1}{2}\sigma^2 w^2 V''(w) + \mu w V'(w) - (\rho + \lambda)V(w) + v(w) = 0. \quad (15)$$

The differential equation (15) is essentially similar to the famous formula derived by Black and Scholes (1973) for option-pricing. This is not surprising, however, because the paper takes the real options approach.

To solve the differential equation (15), let's focus on its homogeneous part:

$$\frac{1}{2}\sigma^2 w^2 V''(w) + \mu w V'(w) - (\rho + \lambda)V(w) = 0. \quad (16)$$

One can confirm by substitution that the solution takes the form Aw^B , where A is a constant to be determined, and B is a root of the quadratic equation:

$$\frac{1}{2}\sigma^2 B(B-1) + \mu B - (\rho + \lambda) = 0.$$

Alternatively, the quadratic equation is written as

$$\frac{1}{2}\sigma^2 B^2 + \left(\mu - \frac{1}{2}\sigma^2\right)B - (\rho + \lambda) = 0. \quad (17)$$

Its discriminant is given by

$$\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2(\rho + \lambda).$$

As both ρ and λ are positive, the discriminant is positive, meaning the existence of two distinctive roots.

Since $(\rho + \lambda)$ is positive on the right-hand-side of equation (17), one of the roots is positive and the other negative. Let B_1 and B_2 denote the positive root and the

negative one, respectively. Then, the general solution to equation (16) can be written as

$$V(w) = A_1 w^{B_1} + A_2 w^{B_2}, \quad (18)$$

where A_1 and A_2 are constants that are yet to be determined. Note that as w approaches to zero, w^{B_2} explodes. Therefore, A_2 must be zero for the boundary condition (13) to be satisfied. Consequently, the general solution is simplified as

$$V(w) = A_1 w^{B_1}, \quad (19)$$

where B_1 is obtained as the positive root of the quadratic equation (17).

To determine A_1 and w^* , another two boundary conditions are required:

$$V(w^*) = \Omega, \quad (20)$$

$$V'(w^*) = 0. \quad (21)$$

The boundary conditions (20) and (21) are called the “value-matching condition” and the “smooth-pasting condition”, respectively. The former says that the value of continuing the life should be equal to that of terminating the one at the threshold w^* . The latter, which is obtained as the derivative of the former, says that the marginal benefit of continuing the life should be equal to that of terminating the one in the optimum.

To find w^* and A_1 from equation (15) with the two boundary conditions (20) and (21), the functional form of v should be specified. Define v as

$$v(w) = \begin{cases} \ln(w) & \text{for } w \geq 1, \\ 0 & \text{for } 0 \leq w < 1. \end{cases} \quad (22)$$

For the range of $w \geq 1$, this function belongs to the class of CRRA utility functions that are widely used in the applied work. Equation (22) means that very low earnings are negligible, which may be contentious. One might suggest that the functional form should be $\ln(w)$ for the whole non-negative range of w , but this violates the boundary condition (13). Alternatively, one might suggest that the functional form should be $\ln(w + 1)$ for the whole non-negative range of w , but this makes it impossible for us to solve the model analytically. As a compromise, the instantaneous indirect utility function is specified as in equation (22).

One can confirm by substitution that the solution to the differential equation (15) takes the form:

$$V(w) = A_1 w^{B_1} + \frac{1}{\rho + \lambda} \ln(w) - \frac{1}{2} \frac{\sigma^2}{(\rho + \lambda)^2} + \frac{\mu}{(\rho + \lambda)^2}. \quad (23)$$

Substitute equation (23) into the boundary conditions (20) and (21). Then, equations (20) and (21) become

$$A_1 w^{*B_1} + \frac{1}{\rho + \lambda} \ln(w^*) - \frac{1}{2} \frac{\sigma^2}{(\rho + \lambda)^2} + \frac{\mu}{(\rho + \lambda)^2} = \Omega, \quad (24)$$

and

$$A_1 B_1 w^{*B_1-1} + \frac{1}{\rho + \lambda} \frac{1}{w^*} = 0, \quad (25)$$

respectively. This set of equations yields

$$\ln(w^*) = (\rho + \lambda)\Omega + \frac{1}{B_1} + \frac{1}{\rho + \lambda} \left(\frac{1}{2}\sigma^2 - \mu \right),$$

which in turn leads to equation (9).

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Notes

¹There are also findings that female suicide rates are significantly responsive to changes in macroeconomic conditions. See, for instance, Chuang and Huang (1997), Jungeilges and Kirchgässner (2002), and Rodríguez Andrés (2005).

²See Cameron (2005) and Yang and Lester (2006) for complete reviews of the literatures on economics of suicide.

³Yang (1987) derives the demand curve for suicide as well as the supply curve of suicide. These curves, however, are not for a market as a whole but for an individual's choice of action.

⁴The original model by Hamermesh and Soss (1974) is more complicated. It takes into account uncertainty about how long an individual can live. For the purpose of this paper, however, such generalisation is unnecessary.

⁵Another modification, which Koo and Cox (2008) make to the Hamermesh and Soss model, is the inclusion of the divorce rate.

⁶See, for instance, Black and Scholes (1973), McDonald and Siegel (1986), Alvarez and Koskela (2005), and Saphores and Boarnet (2006).

⁷See Adda and Cooper (2003, Ch2) for the relationship between the value function and the indirect utility function.

⁸See, for instance, Dixit and Pindyck (1994, Ch4) for how to derive the Bellman equation (7).

⁹A log function belongs to the constant relative risk aversion (CRRA) class of utility functions that are widely used in the applied work. See, for instance, Heijdra and Van Der Ploeg (2002, p. 425) for the explanation about CRRA utility functions.

¹⁰Used are the quarterly data series on the weekly ordinary time earnings of male fulltime workers from ABS catalogue 6302.0. Also used are the monthly data series on the rate of unemployed males looking for fulltime work from

ABS catalogue 6202.0. Difference in occupation among males is ignored, but this may be justified by the Stack (2001) finding that there is little impact of occupation on suicide.

¹¹Suicide cycles are a major focus in the empirical literature on economics of suicide. See, for instance, Koo and Cox (2008), Yang and Lester (1995) and Viren (1996, 2005).

Table 1: Benchmark Values of Parameters

Parameters	Benchmark Value
μ : Expected Value of Wage Growth Rate	0.0100
σ : Standard Deviation of Wage Growth Rate	0.0060
λ : Probability that Wage Drops to Zero	0.0004
ρ : Subjective Discount Rate	0.0100
Ω : Threshold Utility below Which a Person Commits Suicide	1.0000

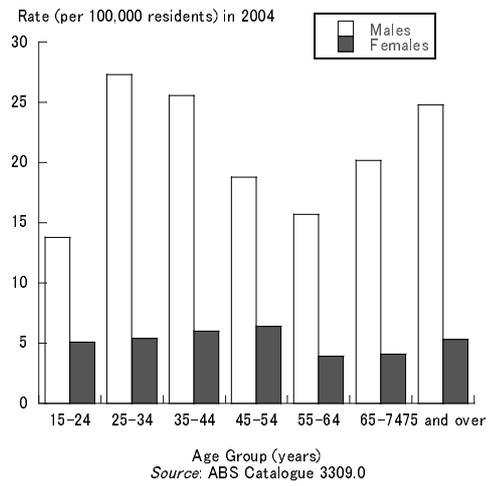
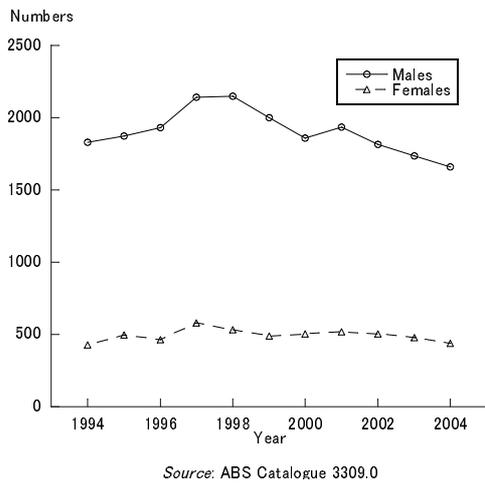


Figure 1: Suicide in Australia

w^* : Threshold Wage below Which an Individual Commits Suicide

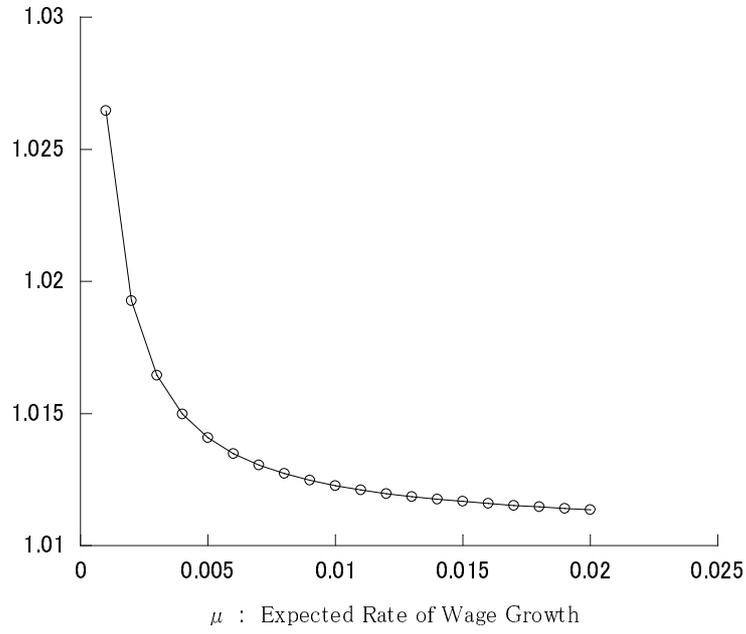


Figure 2: Threshold Wage w^* as a Function of μ

w^* : Threshold Wage below Which an Individual Commits Suicide

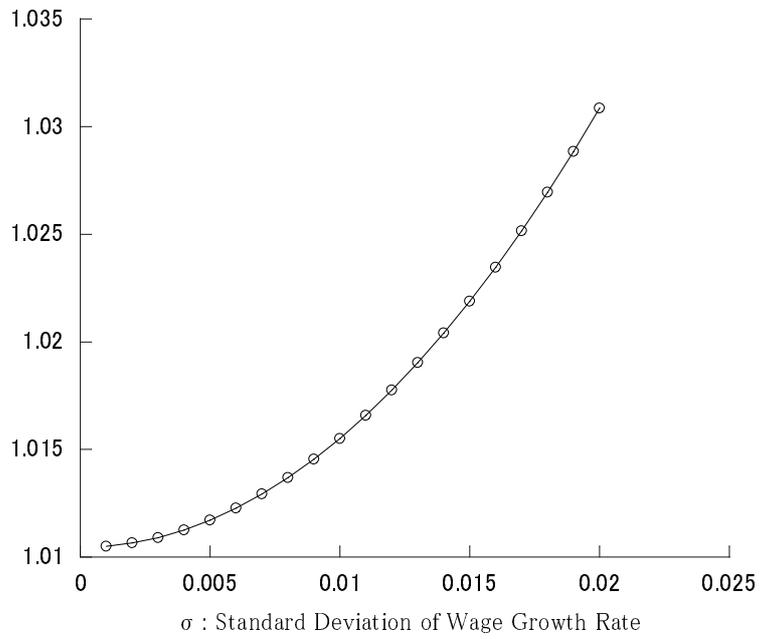


Figure 3: Threshold Wage w^* as a Function of σ

w^* : Threshold Wage below Which an Individual Commits Suicide

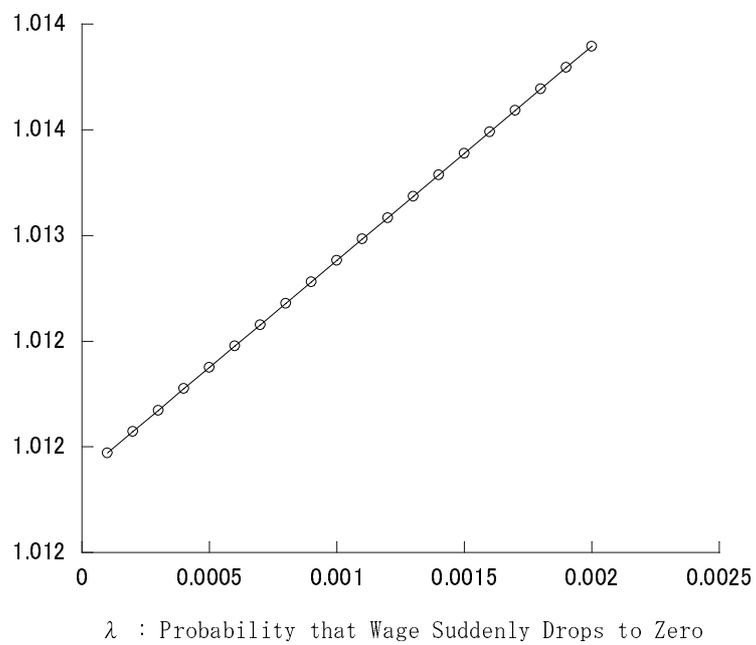


Figure 4: Threshold Wage w^* as a Function of λ