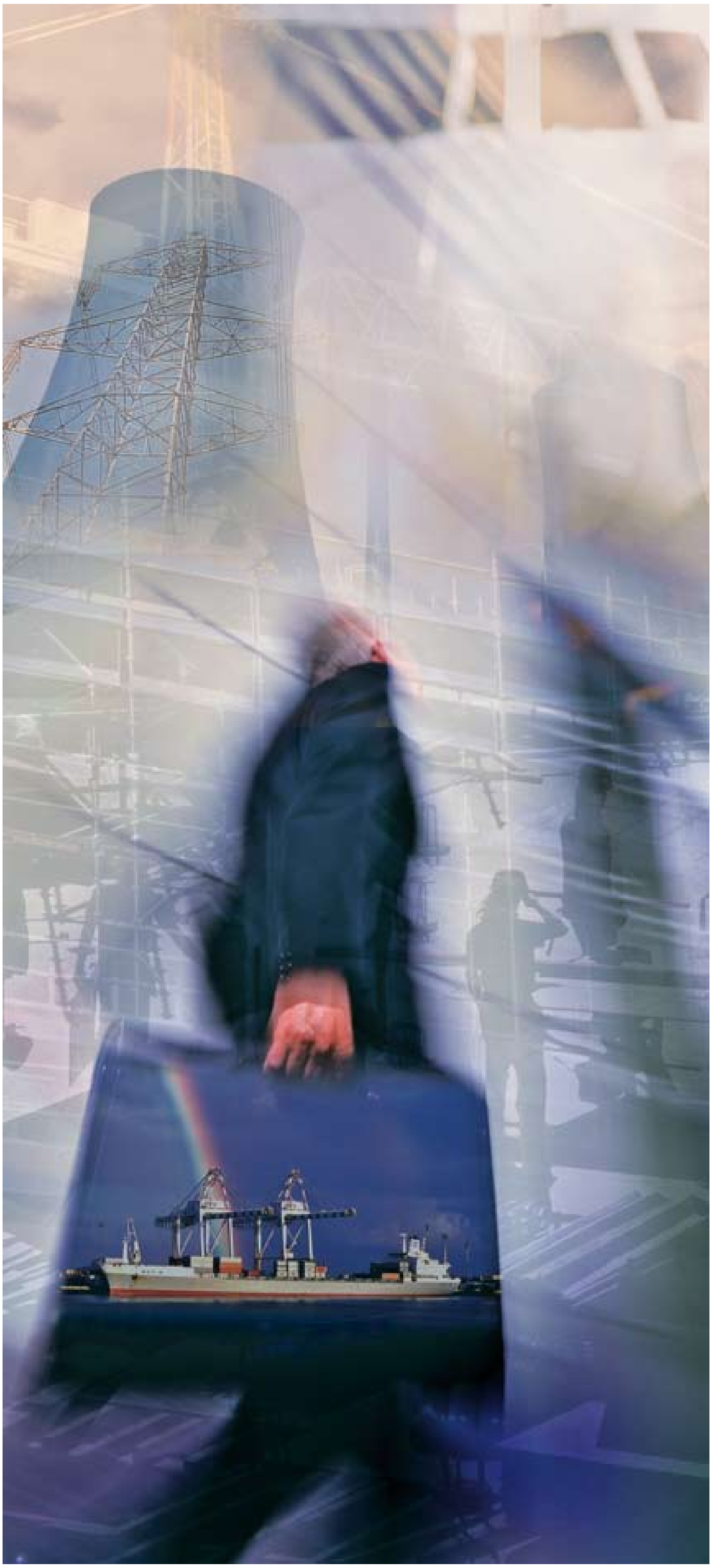

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A Critique of Modelling and Estimating the Effects of Consistent Conditional Variance Expectations on Exchange Rates

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E. J. Wilson

ABSTRACT

A number of authors have recently shown that exchange rate expectations are soundly based on the random walk hypothesis and it is well known the model predicts as good, if not better, than most economic models of exchange rates. This paper suspects that the conditional heteroskedasticity detected in prediction errors of many of these models is primarily due to the reliance on linear first moment, conditional mean representations. In contrast, empirical procedures like GARCH have increased in sophistication to test and correct for the widespread observance of serially correlated second moment, conditional variances in the predictions.

A simple expected utility model which specifies conditional time varying variances is developed to illustrative how empirically correcting for them is a sub optimal research strategy. Simply put, if there remains systematic errors in the model's predictions (other than from trading day, seasonality or similar causes) then it is poor research methodology to empirically remove them. The imbalance in terms of the relative lack of modelling higher order moments and the relatively heavy reliance on removing these effects empirically, needs to be addressed.

Keywords: Random walk, heteroskedasticity, exchange rates.
JEL Classifications: F31, C22, C62, G14.

1. Introduction

The random walk hypothesis has been extensively used to model financial and economic time series over the last fifty years. Its attractiveness is due to its mathematical and conceptual simplicity which hypothesises that successive values of a time series variable, y_t, are independent. That is:

$$y_t = y_{t-1} + v_t \qquad (1)$$

where it is commonly assumed that: $v_t \sim N\left(0,\sigma_v^2\right)$. This means that the change in the variable: $\Delta y_t = y_t - y_{t-1}$ has zero mean and is also independent of past changes. Successive backward substitution of (1) shows that y_t is the sum of all previous shocks: $y_t = \sum_{i=1}^{t} v_{t-i}$, which explains why the process is called a random walk.[1]

The hypothesis has become more popular with its comprehensive application to the analysis of market efficiency, in terms of whether prices and expectations fully reflect all available information and risk. The other major application relates to explaining non-stationary processes, whereby financial and economic shocks have permanent effects, and in finding stationary (cointegrating) relationships and their associated error corrections.

Given the simplicity of the hypothesis and the complex debates it has been used in, it is surprising the random walk has had such success. For example, Meese and Rogoff (1983, p. 3) found that a "random walk model would have predicted major country exchange rates....as well as any of our candidate (economic) models" in terms of out-of-sample predictions. This is striking given the models predictions were calculated using actual future values of the variables, which gave the models a substantial advantage over the random walk model. Burnside *et. al.* (2006) argue that expectations formed with the random walk hypothesis, $y_{t+1}^e = y_t$ are common in the foreign exchange markets. Bacchetta and van Wincoop (2007) agree, especially with carry trade strategies which

[1] This means that y_t will have infinite variance.

typically provide significant excess returns. They reconcile the use of random walk expectations with interest rate parity, provided the foreign exchange portfolios are revised infrequently.

Central to the success of the random walk has been the use of the more general martingale version of the hypothesis. The martingale property only requires that successive changes in a variable are independent of past changes in that variable. This contrasts with the random walk which further requires the conditional higher moments of the change in the variable to be independent. Given the frequent observation of changing volatility in economic and financial markets it has been common practice to allow for (auto)correlated conditional variances.

This has led to parallel developments of formal theoretical economic and financial models and sophisticated estimation processes. Whilst many of the theoretical models are non-linear and stochastic, they have tended to focus on bilinear (multiplicative) processes, regime switching transitions and non-linear dynamics.[2] There has been relatively less emphasis on economic modelling of the higher moments of the probability distributions. On the estimation and testing side, typically, the testing and estimation of time varying conditional variances has been in terms of developing extensions of the basic ARCH model. These include generalised ARCH (GARCH), in mean (GARCH-M), integrated (IGARCH), fractionally integrated (FIGARCH), long-memory (LMGARCH), threshold (TARCH), smooth transition (STRARCH), non-linear (NARCH), exponential (EGARCH), asymmetric (AARCH), quadratic (QARCH), power (PARCH) and component (CGARCH). There are numerous econometric packages which offer many of these procedures with varying distributional assumptions including Gaussian, Student's *t* and Generalised Error Distributions (GED).

Due to the ease of use of these packages, the lack of tractable alternative theoretical models and the inherent time varying distributional properties observed in the time series data, many of these estimation procedures have been widely used. It is argued in this

[2] See Mills and Markellos (2008) excellent surveys in Chapters 5 and 6.

paper that care needs to be exercised because the theoretical models tend to place too much emphasis on the means of the dependent variable (the first moment) whilst the estimation procedures place too much emphasis on correcting for the second moment, conditional variances. A simple illustrative expected utility, exchange rate model is developed in the next section to highlight the possible importance of the variance in utility maximisation. Section 3 demonstrates the difficulty of applying GARCH style specifications to estimate this model, whilst Section 4 provides some conclusions.

2. An Illustrative Model

The basic model assumes wealthholders wish to maximise expected utility:

$$E\left[U(i)\right] = U(r) - R \tag{2}$$

where E is the expectations operator, U is a utility function with the usual properties, i is the total yield of the portfolio, r is the its mean value and R a generalised measure of risk.[3] If a Taylor expansion about i exists for $U(i)$, which is sufficiently large to include all i and if all central moments exist, then:

$$R = \sum_{k=2}^{\infty} \frac{U^k(r)}{k!} - r^k \tag{3}$$

where r^k is the k^{th} central moment of the probability distribution and U^k is the k^{th} derivative of $U(i)$ at r. A possible utility function is the negative exponential:

$$U(i) = -e^{-bi}, \quad b > 0 \tag{4}$$

[3] The following illustration modifies Wilson (1991).

such that: $$R = \left(\frac{b^2\sigma^2}{2!} - \frac{b^3\sigma^3}{3!} - \quad .... \quad \right) e^{-br} . \tag{5}$$

In order to keep this illustrative model simple it will be assumed that order three and above are not significant.[4] It is assumed that wealthholders hold a safe domestic asset, whose one period retum is r_t, and a risky foreign denominated asset, whose one period return is r_t^*. The proportion of the total wealth which is held in foreign assets is denominated by F_t. The total proportional portfolio yield, i_t, is given by:

$$i_t = F_t \left[\left(s_{t+1} + a_t \right) - s_t + r_t^* \left(s_{t+1} + a_t \right) \right] + \left(1 - F_t s_t \right) r_t \tag{6}$$

where s_t is the spot exchange rate (defined as the domestic price of foreign exchange), s_{t+1} the next period spot rate and a_t is a premium reflecting risk and liquidity factors.[5] The portfolio yield given by Equation (6) includes uncertainty in the form of possible capital gain or loss associated with holding foreign assets, $F_t \left(1 + r_t^* \right) s_{t+1}$. Wealthholders therefore choose the proportion of total wealth held in foreign assets F_t which maximises expected utility, for the variance of the spot exchange rate, σ_s^2:

$$F_t \left(s_{t+1}^e - s_t + r_t^* s_{t+1}^e + a_t \right) + \left(1 - F_t s_t \right) r_t - \frac{1}{2} b \left[F_t \left(1 + r_t^* \right) \right]^2 \sigma_s^2 . \tag{7}$$

Differentiating (7) wirh respect to F_t and setting to zero gives the maximum solution:

$$s_t = \lambda \left[a_t + s_{t+1}^e - bF_t \left(1 + r_t^* \right) \sigma_s^2 \right] \tag{8}$$

[4] To the extent this is not be the case then the mean-variance approach only approximates expected utility. Another simplification of this utility function is that it exhibits constant absolute risk aversion. Whilst it is prefered to allow changes in wealth to affect the behaviour of wealtholders, the model does not provide tractable results.

[5] The premium can also include other factors like brokerage costs and withholding taxes.

where: $\lambda = \left(1+r_t^*\right)/\left(1+r_t\right)$ represents the ratio of relative discount factors.

Now in order to obtain the rational expectations solution, first expand the deterministic λ ratio to include an additive stochastic component, $u_{\lambda,t} \sim N\left(0, \sigma_{u_\lambda}^2\right)$ along the lines of Begg (1984). Second, simplify (8) by assuming the proportion of foreign assets and risk premium are fixed at F and a respectively. Finally, assume that wealtholders, under expected utility, form rational expectations of the mean, $s_{t,t+1}^e$ and variance, $\sigma_{t,t+1}^{2e}$ of the future spot exchange rate, based on all the available information, Ω_t. Taking conditional expectations of (8) and subtracting from (8) gives: $s_t - s_{t,t+1}^e = s_{t,t+1}^e\left(u_{\lambda,t+1}/\lambda\right)$. Squaring and taking conditional expectations derives, $\sigma_{t,t+1}^{2e} = \left(s_{t,t+1}^e\right)^2\left(\sigma_{u_\lambda}^2/\lambda^2\right)$, which when substituted into (8) gives the required relationship:

$$s_t^e = a\lambda + \lambda s_{t,t+1}^e - \gamma\left(s_{t,t+1}^e\right)^2 \qquad (9)$$

where: $\gamma = bF\left(1+r_t^*\right)\sigma_{u_\lambda}^2/\lambda^2$. This describes the rational expectations forward evolution of the spot exchange rate. The quadratic characteristics of the next period spot rate are due to the taking of rational expectations of the second moment of the distribution of future exchange rates.[6] According to (9), an expected future depreciation will lead to a current period depreciation, according to the size of λ. Remember that, $\lambda = \left[\left(1+r_t^*\right)/\left(1+r_t\right)\right] - u_{\lambda,t}$ with $u_{\lambda,t} \sim N\left(0, \sigma_{u_\lambda}^2\right)$, which implies that λ will centre around unity, consistent with r_t and r_t^* being cointegrated (according to the uncovered interest parity condition). It can also be seen that an increase in the overseas interest rate, r_t^* relative to the domestic rate, r_t will cause the current spot rate to depreciate.

The possible dynamics are important and will be furthered explored in the next section after considering problems with estimating the unknown model using GARCH methods.

[6] If the higher moments in (5) were included then (9) would comprise a higher order polynomial.

3. Random Walks and Heteroskedastic Corrections

Transforming (9) and simplifying the notation gives the simple autoregressive representation:

$$s_{t+1} = \beta_0 + \beta_1 s_t + v_t \tag{10}$$

with $\beta_0 = -a$, $\beta_1 = 1/\lambda \propto (1+r_t)/(1+r_t^*)$ and $v_t = (\gamma/\lambda) s_{t+1}^2$. Ignoring the important problem with the error term for the moment, if $\beta_1 = 1$ then we have:

$$\Delta \mathrm{s}_{\mathrm{t+1}} = \beta_0 + \beta_0' \left(f_t^1 - s_t \right) + \upsilon_t .$$

Sarno and Taylor (2002) provide a comprehensive review of the difficulties associated with empirically testing this specification, including many of its variations. However, if $\beta_1 = 1$ then considering (10) as a random walk with drift would be wrong because of the invalid assumption about v_t.[7] Now if (10) is estimated, incorrectly assuming a GARCH(1,1): $\sigma_t^2 = \alpha_0 + \alpha_1 v_{t-1}^2 + \alpha_2 \sigma_{t-1}^2$ process, the Gaussian likelihood function will be:

$$l_t = -\frac{1}{2} log(2\pi) - \frac{1}{2} log\, \sigma_t^2 - \frac{1}{2}\left(s_{t+1} - \beta_0 - \beta_1 s_t\right)^2 / \sigma_t^2 . \tag{11}$$

The estimates, $\hat{\beta}_0$ and $\hat{\beta}_1$ will be biased due to the exclusion of s_{t+1}^2 in (10) and (11). However the misspecification will be even greater because excluding s_{t+1}^2 removes potentially interesting and important dynamics. If instead, the researcher selected the GARCH-M process (out of the many possible configurations, especially when the true model is unknown) then this would be better than GARCH. This is because (8) would effectively be estimated, since it is (9) with the substitutions, $\sigma_{t+1}^2 = s_{t+1}^2 \left(\sigma_{u_\lambda}^2 / \lambda^2 \right)$ and

[7] If the risk premium, a, is zero then it as a simple random walk.

$\delta = bF\left(1 + r_t^*\right)$:

$$s_{t+1} = \beta_0 + \beta_1 s_t + \delta\sigma_{t+1}^2 + \varepsilon_t .$$

The likelihood function:

$$l_t = -\frac{1}{2}log\left(2\pi\right) - \frac{1}{2}log\,\sigma_{\varepsilon_t}^2 - \frac{1}{2}\left(s_{t+1} - \beta_0 - \beta_1 s_t - \delta\sigma_{t+1}^2\right)^2 \Big/ \sigma_{\varepsilon_t}^2$$

will estimate $\hat{\delta}$ although δ differs from γ by the amount $\sigma_{u_\lambda}^2 / \lambda^2$. If the researcher new the correct model then it is possible to determine $\hat{\lambda}$ from $\hat{\beta}_1 = 1/\hat{\lambda}$, although unbiasedness will not carry through the non-linear transformation and there is the extra complication that $\sigma_{u_\lambda}^2$ is not known.

It is therefore likely that even with the most appropriate GARCH-M estimation specification, γ will not be correctly estimated. This is important because small changes in parameters in this type of dynamic models can lead to significant consequences for the stability of the system and the dynamic solution to, or from, steady state. Solving equation (9) by completing the square, for the steady state, $\overline{s}$:

$$\overline{s} = \frac{\lambda + \left[\left(\lambda - 1\right)^2 + 4a\lambda^2\gamma\right]^{\frac{1}{2}}}{2\lambda\gamma} \tag{12}$$

shows that an increase in the domestic interest rate, *r* (decrease in r^*) will decrease λ and therefore appreciate the steady state exchange rate. Similarly, an increase in risk aversion, *b* and interest rate uncertainty, $\sigma_{u_\lambda}^2$ will increase γ, which will cause $\overline{s}$ to appreciate.

The stability properties of the steady state, $\overline{s}$ can be determined by taking the derivative, $\overline{s}' = \partial s_t^e / \partial s_{t,t+1}^e$, of (9) to give:

$$\bar{s}' = 1 - \left[(\lambda - 1)^2 + 4a\lambda^2\gamma \right]^{\frac{1}{2}} < 1 . \tag{13}$$

When $0 < \bar{s}' < 1$ Equation (13) describes the forward evolution of rational expectations in (9) as taking a monotonic convergent path. The forward looking wealtholders will act accordingly so the economy will move to the to the steady state, $\bar{s}$. However, if $-1 < \bar{s}' < 0$, the forward evolution of expectations specified in (9) will form an explosive spiral, which will not realise a steady state. Wealtholders will therefore have to back track, setting the current level of s_t as the next period expectation. This effectively reverses the direction specified in (9) to:

$$s^e_{t+1} = a\lambda + \lambda s_t - \gamma s_t^2 . \tag{9'}$$

Equation (13) also shows that an increase in the risk premium, *a*, risk aversion, *b*, interest rate uncertainty, $\sigma^2_{u_\lambda}$ or proportion of wealth held in the risky foreign asset, *F* will increase the need for this back tracking process.

Unfortunately the dynamic system becomes more complicated for values of $\bar{s}' < -1$ and it is necessary to use van der Ploeg's (1985) transformation of (9) which gives the logistic equation:

$$x_{t+1} = \theta x_t - \theta x_t^2 \tag{14}$$

where the steady state relationship is defined as: $\bar{x} = (\gamma\bar{s} - \omega) / 1 - 2\omega$ and $\theta = \lambda(1 - 2\omega)$ with $\omega = \left\{ (\lambda - 1) - \left[(\lambda - 1)^2 + 4a\lambda^2\gamma \right]^{\frac{1}{2}} \right\} \Big/ 2\lambda$. Equation (14) clearly shows the reverse behaviour required for values, $2 < \theta < 3$, that is, $-1 < \bar{s}' < 0$. The value of θ is constrained to the range $1 < \theta < 4$ which coincides with $-2 < \bar{s}' < 1$. For the values, $3 < \theta < 3.828$ which is equivalent to, $-1.828 < \bar{s}' < -1$, there are increasing even, then odd period cycles. When $3.828 < \theta < 4$ and $-2 < \bar{s}' < -1.828$, the system becomes chaotic and will exhibit speculative bubble behaviour.

This complication has not been introduced to discuss chaos, rather it is to highlight the possible sensitivity of the deterministic dynamic system to parameter values. Accordingly inappropriate estimation techniques which obtain biased parameter estimates may have significant consequences for model predictions and policy prescriptions.

4. Conclusion

Whilst there have been some recent theoretical developments, economic models of exchange rates generally do not out perform the random walk. This paper suspects that the conditional heteroskedasticity detected in prediction errors of many of these models is primarily due to the reliance on linear first moment, conditional mean representations. In contrast, empirical procedures have increased in sophistication to test and correct for the widespread observed occurrence of serially correlated second moment, conditional variances.

There has been some developments of non-linear exchange rate models, although they do not give a central role to higher moments of the data generating process. This would appear to be the legacy from the dominance over decades, of the rational expectations literature. It is understandable why the multitude of GARCH techniques have been developed as a response, however it is important to acknowledge that empirical correction for misspecified models is a sub optimal research strategy. Simply put, if there remains systematic errors in the model's predictions (other than from trading day, seasonality or similar causes) then it is poor research methodology to just remove them in the estimation process. There is an imbalance, in terms of the relative lack of modelling higher order moments and the relatively heavy reliance on removing these effects empirically, which needs to be addressed.

A simple illustrative expected utility, exchange rate model was presented to demonstrate the importance of serially dependent time varying conditional variances. It was shown that inappropriately ignoring this effect and empirically correcting for it, provided biased parameter estimates. The problem is that the dynamic non-linear model is sensitive to relatively small differences in estimated parameter values, which has important consequences for the stability of the steady and the dynamic paths of adjustment. Unfortunately there have not been many studies which have attempted to examine the non-linear effects of higher moments. Diebold and Nason (1990) explore this nonparametrically although they were not able to detect any patterns which enable improvements in model predictions. However it may be due to the more complicated nature of non-linear models and more work needs to be done.

The phenomena of time varying conditional variances have more recently led to the development of stochastic volatility models. Whilst the empirical work has lagged due to the lack of closed form analytic solutions in these complicated models, recent computing advances are now providing numerical methods. These models can be specified in discrete and continuous time which may lead to further theoretical economic and financial developments.

Bibliography

Baccetta, P. and E. van Wincoop (2007), "Random Walk Expectations and the Forward Discount Puzzle", *American Economic Review*, 97, May, pp. 346-350.

Begg, D. (1984), "Rational Expectations and Bond Pricing: Modelling the Term Structure With and Without Certainty Equivalence", *Economic Journal Supplement*, 94, pp. 45-58.

Burnside, C., Eichenbaum, M., Kleshchelski, I. and S. Rebelo (2006), "The Returns to Currency Speculation", *NBER Working Paper* No. 12489.

Diebold, F and J. Nason (1990), Nonparametric Exchange Rate Prediction", *Journal of International Economics*, 28, pp. 315-332.

Flood, R. and R. Hodrick (1986), "Asset Price Volatility, Bubbles and Process Switching", *The Journal of Finance*, 61, pp. 831-842.

Frankel, J. and K. Foot (l987), "Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations", *American Economic Review*, 77, pp. 133-153.

Hansen, L. and R. Hodrick (1980), "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis", *Journal of Political Economy*, 88, pp. 829-853.

Meese R. and K. Rogoff (1983), "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?", *Journal of International Economics*, 14, pp. 3-24.

Meles, F. and W. Transue (1986), "Unscrambling Chaos Through Thick and Thin", *The Quarterly Journal of Economics*, 101, pp. 419-423.

Mills, T. and R. Markellos (2008), *The Econometric Modelling of Financial Time Series*, 3rd ed., Cambridge University Press, Cambridge.

Obstfeld, M. and K. Rogoff (1986), "Ruling Out Nonstationary Speculative Bubbles", *Journal of Monetary Economics*, pp. 101-121.

Sarno, L. and M. Taylor (2002), *The Economics of Exchange Rates*, Cambridge University Press, Cambridge.

van der Ploeg, F. (1985), "Rational Expectations, Risk and Chaos in Financial Markets", *The Economic Journal Conference Papers Supplement*, pp. 151-162.

Wilson, E. J. (1991), "Modelling the Effects of Consistent Conditional Variance Expectations on Short-Run Exchange Rate Behaviour", *Midsouth Academy of Economics and Finance Conference Proceedings*, Shreveport, Louisiana, 6-9 February, pp. 597-602.