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# Redistributive Policies and TFP Differences Across Countries

BY

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Economists point towards cross-country differences in TFP to explain wide differences in income per capita across countries. This paper derives theoretical restrictions for interpreting the data based on a neoclassical, dynamic general equilibrium model in which redistributive policies significantly alters overall productivity as measured by the conventional definition of TFP. The numerical simulations of the model's outcome highlights that whether or not the cross-country differences in output per capita can be significantly explained by the TFP differences depends on the sources of TFP differences. If in the sample of countries the TFP differences arise mainly due to institutional and demographic factors then input differences would be more significant than TFP differences in explaining differences in output per capita. If, however, the TFP differences arise mainly from differences in the degree of redistribution then the TFP differences would be more significant than input differences.

KEYWORDS: heterogeneous agents, edogenous TFP, neighborhood externality, progressive redistribution, macro gains versus micro losses.

*"Gaining a quantitative understanding of ...forces for diffusion - and, .... the forces that oppose them - is the central question of the theory of economic growth and development"* - Robert E. Lucas, Jr. (2000).

## 1 Introduction

Solow (1957) defines for his model the total factor productivity of TFP as the ratio of output to an weighted average of input based on a neoclassical production function. Following that definition, economists identify TFP as the residual of a regression of output on a combination of inputs. Consequently, TFP is conventionatelly labelled as the Solow residual.

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In other words, it reflects the portion of real output which is not accounted for by inputs of labour and capital. Some people interpret it as the overall productivity with which inputs are transformed into outputs. This overall productivity could be affected by many factors, such as technology, externalities, unaccounted inputs, resource reallocations and obviously measurement errors. To the extent economic policies affect any of these determinants of TFP it is likely to have an impact on the measured TFP as well. We argue that redistributive policy have significant implications for the TFP of a large number of countries suffering from ill developed credit markets. Few models have been written to address this issue. In this paper, therefore, we first present a dynamic general equilibrium model to understand how redistributive policies affect the growth of inputs and TFP and then simulate the model outcome to understand the relative importance of the variation of TFP and input in explaining variation of output per capita.

Baier, Dwyer and Tamura (2006) report considerable heterogeneity among countries regarding how much of the variation of growth of output per worker can be explained by the variation of growth of physical and human capital per worker vis-à-vis the variation of the growth of TFP. Todaro and Smith (2006) conclude that differences in physical and human capitals per capita are the key to explaining the differences of income per capita. They claim that the essential factors for development are natural resources and skilled human resources who have access to critical markets. The economy could develop only when a large number of skilled citizens can use sufficient resources to produce outputs. Therefore, the gap of income between rich and poor countries is mainly due to accumulation of physical and human capital. Prescott (1998) argues, however, that even where the stock of physical and human capital are similar, total outputs would be significantly different if countries have significantly different levels of TFP. Others such as Hall and Jones 1996, 1999, Klenow and Rodriguez-Clare 1997, Aiyar and Feyrer 2001 and Easterly and Levine 2001 supports that argument. Klenow and Rodriguez-Clare (1997) estimate that about 90 percent of the variation in growth rate of output per worker for a sample of 98 countries during 1960–85 could be accounted for by the differences in TFP growth after considering human capital. They also find that productivity differences may account for 56% of differences in income per worker. Aiyar and Feyrer (2001) test the links between human capital accumulation and growth in TFP and find that human capital accumulation is a crucial determinant of the dynamic path of TFP, and that TFP differences explain most of the static variation in GDP across countries.

The above models rely on empirical analysis to support their argument. Prescott (1998) calls for a theory of TFP to interpret these empirics. Bandyopadhyay (2004) provides a the-

ory of TFP suitable for empirical analysis based on a dynamic general equilibrium model of economic growth as well as an explicit formula for TFP as a function of a redistributive fiscal policy and the distribution of human capital. In this paper, we present another theory of TFP which, unlike Bandyopadhyay (2004), can be used to distinguish clearly four alternative sources of TFP variations: institutions, demography, technology and policy regimes for redistribution. We apply this theory of TFP to identify some conditions to characterize when TFP differences may significantly explain differences in income per capita and when it may not.

Most countries pursue economic policies to achieve some degree of redistribution. Benabou (2002) and others show that those policies could also have significant implications for economic growth. We argue that those policies have important effects on the measured TFP of a country. To understand the relative importance of TFP variations in explaining cross-country disparities of per capita income we, therefore, focus on a model where the TFP for the economy varies endogenously as a function of country-specific characteristics and, in particular, its redistributive policy. In our model, incomplete markets restrict parental investments in children of heterogeneous ability to a fraction of their disposable income and that gives rise to inefficiently rigid interpersonal differences in productivity. In presence of a convex technology such disparity provides scopes for a redistributive policy to generate some gains in TFP.

The parameters that affect the model's outcome of TFP include two institutional parameters, which capture the degree of social segregation and the quality of the education system, two technology parameters, which measures the output elasticity of two types of capital inputs, a demographic parameter to denote the extent of talent diversity in the population and, a policy parameter, which identifies the degree of redistribution under a progressive income tax regime. The demography of the population determines the variance of inborn ability that affects the distribution of human capital and the TFP. The degree of segregation affects the overall productivity since it captures the adverse effect on TFP due to the barriers to interpersonal communication and knowledge spillover. The quality of education affects overall productivity because it directly affects the effectiveness of investment in human capital.

We apply this theory of TFP to estimate how much the differences of per capita income across economies can be explained by differences in inputs and TFP. By calculating elasticity of per capita income, TFP and inputs with respect to institutional and policy parameters, we find that TFP differences significantly explain per capita income differences across countries if the source of TFP differences is policy and not institution.

Following the introduction, section 2 describes the model and gives analytical expressions of TFP. Section 3 discusses alternative sources of variations of TFP and answers the question of how much the differences of income across countries can be explained by differences in TFP. Section 4 highlights that understanding sources of the variations provides an important key to understand how TFP differences explain differences in per capita income. We attach the Appendix including proofs of lemmas and propositions that are not included in the paper following the list of references.

## 2 The Model

### 2.1 Preference, Technology and Endowments

The model follows Kreps and Porteus (1979), Louri (1981), Epstein and Zin (1989) and Benabou (2002). It considers a continuum of infinitely lived dynasties  $i \in [0, 1]$ . Each dynasty is made of a sequence of families consisting of individuals who live for two periods or two generations, first as a child and then as an adult. In each period  $t$ , the dynasty is represented by a family of an adult and a child. The adult, in period  $t$  represents the dynasty from that period onward and makes all decisions for that period subject to the constraint that she cannot pass on her debt to her child. We call this adult of the dynasty  $i$  in period  $t$  the dynastical agent  $i$  or simply agent  $i$ . The preference of the dynastical agent  $i$  at period  $t$  is given by:

$$(1) \quad \ln U_t^i = E_t \left[ \sum_{n=0}^{\infty} \rho^n (\ln c_{t+n}^i - (l_{t+n}^i)^\eta) \right], \eta > 1,$$

where  $c_t^i \geq 0$  and  $l_t^i \in [0, 1]$  denote, respectively, consumption and labor supply by the adult of the dynasty  $i$  in period  $t$ ;  $\rho \in (0, 1)$  is the discount factor. To describe the production process, we consider two cases.

We assume that everyone operates a technology which allows both physical and human capital to affect output as complementary inputs in the same way as Barro, Mankiw and Sala-i-Martin (1995) such that the output of the self-employed agent  $i$  as a function of her physical and human capital  $k_t^i, h_t^i$  and labor  $l_t^i$  satisfies

$$(2) \quad y_t^i = (k_t^i)^\lambda (h_t^i)^\mu (l_t^i)^\varepsilon, \text{ where, } \varepsilon = 1 - \lambda - \mu.$$

The government has a scheme of progressive income taxation and transfer. In general, it may also provide an education subsidy and a bequest subsidy to offset the negative effects of income tax on output and finance those subsidies by taxing consumption at a rate  $\theta \in (0, 1)$  per unit.<sup>3</sup> As in Benabou (2002), we assume that agents cannot inherit debt from the previous generations. Nor can they pass on debt to their children. Consequently, at each date, the disposable income  $\hat{y}_t^i$  of the agent  $i$  must equal the total expenditure on consumption  $c_t^i$ , consumption tax  $\theta c_t^i$ , private education expenditure  $e_t^i$  and bequest  $b_t^i$ . In other words,

$$(3) \quad \hat{y}_t^i = (1 + \theta) c_t^i + e_t^i + b_t^i.$$

The agent receives education subsidy at a rate  $d \in (0, 1)$  per unit of her expenditure  $e_t^i$  on the child's education such that in the following period her grown up child's human capital  $h_{t+1}^i$  as a function of her innate ability  $\xi_{t+1}^i$ , external effects arising from neighborhood or family as proxied by parental human capital  $h_t^i$ , and the sum of private and public investment on her education  $(1 + d) e_t^i$ , is given by,

$$(4) \quad h_{t+1}^i = \kappa \xi_{t+1}^i (h_t^i)^\alpha ((1 + d) e_t^i)^\beta.$$

The idiosyncratic shocks  $\xi_t^i$  that arise from discrepancies in innate ability or in efficiency of human capital usage are *i.i.d.* with  $\ln \xi_t^i \sim N(\varphi, \sigma^2)$ , where  $\varphi$  and  $\sigma^2$  are constants. The parameter  $\alpha \in (0, 1)$  measures the child's human capital elasticity of "neighborhood externality," a phrase explored originally in Benabou (1996) in the context of human capital inequality and the parameter  $\beta \in (0, 1)$  measures the same elasticity of the education expenditure, which is primarily determined by the quality of the education system.

Capital goods are complementary to human capital and become obsolete at the end of each generation. A tool loses value when its user dies. Parents buy new tools for their children at a subsidized rate set by the government and leave them as bequest. To capture this feature we assume that they depreciate completely in the production process. Consequently, in the generation  $t + 1$ , the agent  $i$ 's physical capital  $k_{t+1}^i$  consists only of her parent's bequest  $b_t^i$  and a bequest subsidy from the government at the rate of  $v \in (0, 1)$  per unit of the bequest such that

$$(5) \quad k_{t+1}^i = (1 + v) b_t^i.$$

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<sup>3</sup>We could regard bequest subsidy as a kind of subsidy to the purchase of capital goods or an exemption for estate taxation.

Initial endowments of physical and human capital  $k_0^i$  and  $h_0^i$  are jointly, lognormally distributed and the adult receives one unit of labor endowment in each period.

## 2.2 Redistribution with Progressive Income Tax

By assumption, the government cannot detect individual innate ability  $\xi_t^i$  and neighborhood or family effects  $h_t^i$ , but does observe individual incomes  $y_t^i$  and their expenditure on education  $e_t^i$ . Following Benabou (2002), the disposable income of a typical agent at a date  $t$  satisfies

$$(6) \quad \hat{y}_t^i \equiv (y_t^i)^{1-\tau} (\tilde{y}_t)^\tau,$$

such that those with income higher than  $\tilde{y}_t$  pay net tax while those with income below  $\tilde{y}_t$  receive net transfers and the balanced-budget constraint is

$$(7) \quad \int_0^1 (y_t^i)^{1-\tau} (\tilde{y}_t)^\tau di = y_t,$$

where  $y_t \equiv \int_0^1 y_t^i di$  denotes per-capita income,  $\tilde{y}_t$  represents the break-even level of income and  $0 < \tau < 1$  measures the average marginal tax rate and is identified as the degree of redistribution or *progressivity* in fiscal policy. Note that on a logarithmic scale  $\tau$  denotes the proportional tax rate on the log of personal income and we only focus on redistributive policies that transfer resources only from high income to low income people.

## 2.3 Individual Optimization

To get our main points across easily we focus on the stationary policy sequence,  $T \equiv (\tau, d, v, \theta)$ . At each date  $t$ , let  $m_{ht}, m_{kt}$  denote the means and  $\Delta_{ht}^2, \Delta_{kt}^2$  denote the variances of  $\ln h_t^i$  and  $\ln k_t^i$ , respectively, and let  $cov_t$  denote the covariance between  $\ln h_t^i$  and  $\ln k_t^i$ . Suppose  $M_t \equiv (m_{ht}, m_{kt}, \Delta_{ht}^2, \Delta_{kt}^2, cov_t)$ . Then for the agent's dynamic optimization problem, the state variables are  $(h_t^i, k_t^i, M_t; T)$ , the control variables are  $(c_t^i, l_t^i, e_t^i, b_t^i)$  and the Bellman equation is as follows

$$(8) \quad \ln U(h_t^i, k_t^i, M_t; T) = \max_{c_t^i, l_t^i, e_t^i, b_t^i} \left\{ (1 - \rho) [\ln c_t^i - (l_t^i)^\eta] + \rho E_t [\ln U(h_{t+1}^i, k_{t+1}^i, M_{t+1}; T)] \right\},$$



subject to (2), (3), (4), (5) and (6).

Lemma 1: *The value function under fiscal redistribution is  $\ln U (h_t^i, k_t^i, M_t; T) = Z_1 (\ln h_t^i - m_{ht}) + Z_2 (\ln k_t^i - m_{kt}) + W_t$ , where*

$$(9) \quad Z_1 = \frac{(1 - \rho)\mu(1 - \tau)}{(1 - \rho\alpha)(1 - \rho\lambda(1 - \tau)) - \rho\beta\mu(1 - \tau)},$$

$$(10) \quad Z_2 = \frac{(1 - \rho\alpha)(1 - \rho)\lambda(1 - \tau)}{(1 - \rho\alpha)(1 - \rho\lambda(1 - \tau)) - \rho\beta\mu(1 - \tau)},$$

and aggregate welfare is  $W_t = \int_0^1 \ln U (h_t^i, k_t^i, M_t; T) di$ .

The values of human capital and physical capital as expressed by their utility elasticities are respectively given by  $Z_1$  and  $Z_2$ . Note the tax rate  $\tau$  can alter these values individually but does not alter the relative value of human to physical capital,  $\frac{\mu}{\lambda(1-\rho\alpha)}$ , which increases with output elasticity of human capital  $\mu$ , neighborhood effect  $\alpha$  and patience  $\rho$  but remains unaffected by the quality of education  $\beta$ .

## 2.4 Labor Supply and Savings Decisions

The first order conditions associated with the Bellman equation described by (8) yield complete solutions to the agent's problem. We first discuss labor supply followed by investments in education and bequest.

Lemma 2: *The optimal labor supply remains invariant to time and personal characteristics and decreases with the average marginal income tax rate  $\tau$  such that:*

$$(11) \quad l_t^i = l \equiv \left( \frac{((1 - \lambda - \mu)/\eta)(1 - \rho\alpha)(1 - \tau)}{(1 - \rho\alpha)(1 - \rho\lambda(1 - \tau)) - \rho\beta\mu(1 - \tau)} \right)^{1/\eta}.$$

Next we consider investment propensities for the two forms of capital. We denote by  $s_{jt}^i$ ,  $j = 1, 2$ , respectively the fraction of disposable income that agent  $i$  invests in her children's education and for her bequest such that  $s_{1t}^i \equiv e_t^i/\hat{y}_t^i$ ,  $s_{2t}^i \equiv b_t^i/\hat{y}_t^i$ .

Lemma 3: *The education investment rate  $s_{1t}^i$  and the bequest rate  $s_{2t}^i$  are time invariant*

and decrease with the average marginal income tax rate  $\tau$ :

$$(12) \quad s_{1t}^i = s_1 \equiv \frac{\rho\beta\mu(1-\tau)}{1-\rho\alpha} \equiv (1-\tau)\bar{s}_1,$$

$$(13) \quad s_{2t}^i = s_2 \equiv \rho\lambda(1-\tau) \equiv (1-\tau)\bar{s}_2,$$

where  $\bar{s}_1 = \frac{\rho\beta\mu}{1-\rho\alpha}$  and  $\bar{s}_2 = \rho\lambda$  denote the laissez-faire saving rates.

From (12) and (13) we note that the relative propensity of investment between human and physical capital increases with the quality of education  $\beta$  as well as all other factors that raise the relative utility valuation of human to physical capital. Lemmas 2 and 3 spell out explicitly the negative effect of redistribution on the incentives to supply labor and capital inputs.

## 2.5 Consumption Taxes, Education and Bequest Subsidies

Benabou (2002) emphasizes that elected governments do use a wide range of instruments rather than mere income taxation to redistribute resources among people and across time. Typically governments attempt to offset some of the distortionary effects of income taxes with a package of redistributive policies. In particular, we assume that the government chooses the subsidy rates  $d$  and  $v$  such that:

$$(14) \quad (1+d)s_1 = \bar{s}_1,$$

$$(15) \quad (1+v)s_2 = \bar{s}_2.$$

It finances those subsidies with consumption tax by setting the tax rate  $\theta$  such that

$$(16) \quad \theta \int_0^1 c_t^i di = d \int_0^1 e_t^i di + v \int_0^1 b_t^i di.$$

By the government budget constraint (16) and by Lemma 3, it follows that

$$(17) \quad \frac{\theta(1-s_1-s_2)}{1+\theta} = ds_1 + vs_2,$$

and by (14), (15) and (17), the subsidy rates  $d$  and  $v$  and the consumption tax rate  $\theta$  satisfy,

$$(18) \quad d = \frac{\tau}{1 - \tau}, v = \frac{\tau}{1 - \tau} \text{ and } \theta = \frac{\bar{s}_1 + \bar{s}_2}{1 - \bar{s}_1 - \bar{s}_2} \tau.$$

We can switch on the intertemporal distortions simply by setting either  $d$  or  $v$  or both equal to zero and by adjusting  $\theta$  according to (17). By (18) the redistributive policy package can be represented by the parameter  $\tau$  alone. To capture the breadth of redistribution, therefore, we refer to  $\tau$  as the degree of redistribution under the income tax regime rather than just the average marginal tax rate.

We now describe the equilibrium dynamics before going to the section on steady state and the section containing key proposition of this paper.

### 3 The Equilibrium Dynamics

The optimization problem (8) yields (11)-(13) and other decision rules as follows:

$$(19) \quad \ln c_t^i = \ln(1 - s_1 - s_2) - \ln(1 + \theta) + (1 - \tau) \ln y_t^i + \tau \ln \tilde{y}_t,$$

$$(20) \quad \ln e_t^i = \ln s_1 + (1 - \tau) \ln y_t^i + \tau \ln \tilde{y}_t,$$

$$(21) \quad \ln b_t^i = \ln s_2 + (1 - \tau) \ln y_t^i + \tau \ln \tilde{y}_t.$$

Together with the government's budget constraints (7) and (16) the above decision rules imply a unique sequence of aggregate state variables  $\{M_t\}$  that coincides with what the agent  $i$  takes as given in (8) such that at each date  $t = 0, 1, 2, \dots$ , the following aggregate consistency condition holds:

$$(22) \quad \int_0^1 y_t^i di = \int_0^1 c_t^i di + \int_0^1 e_t^i di + d \int_0^1 e_t^i di + \int_0^1 b_t^i di + v \int_0^1 b_t^i di.$$

### 3.1 *Dynamic Path of Physical Capital, Human Capital and Income with BT*

The logarithm of (5), combining with (11) and (13) yields the dynamics of physical capital for the dynasty  $i$ ,

$$(23) \quad \ln k_{t+1}^i = \ln \bar{s}_2 + (1 - \lambda - \mu)(1 - \tau) \ln l + \lambda(1 - \tau) \ln k_t^i + \mu(1 - \tau) \ln h_t^i + \tau \ln \tilde{y}_t.$$

The logarithm of (4), combining with (11) and (12) yields,

$$(24) \quad \ln h_{t+1}^i = \ln \kappa + \beta \ln \bar{s}_1 + \beta(1 - \lambda - \mu)(1 - \tau) \ln l + \ln \xi_{t+1}^i + \beta\lambda(1 - \tau) \ln k_t^i + (\alpha + \beta\mu(1 - \tau)) \ln h_t^i + \beta\tau \ln \tilde{y}_t.$$

Substituting (23) and (24) into (2) yields the equilibrium path of income for agent  $i$ :

$$(25) \quad \ln y_{t+1}^i = \psi + (1 - \alpha)(1 - \lambda - \mu) \ln l + \mu \ln \xi_{t+1}^i + (\lambda + \beta\mu) \tau \ln \tilde{y}_t - \alpha\lambda\tau \ln \tilde{y}_{t-1} + (\alpha + (\lambda + \beta\mu)(1 - \tau)) \ln y_t^i - \alpha\lambda(1 - \tau) \ln y_{t-1}^i,$$

where  $\psi = \mu(\ln \kappa + \beta \ln \bar{s}_1) + \lambda(1 - \alpha) \ln \bar{s}_2$  is a constant.

Note that the intergenerational persistence of human capital  $p^h \equiv \alpha + \beta\mu(1 - \tau)$  and physical capital  $p^k \equiv \lambda(1 - \tau)$  together imply the intergenerational persistence of income  $p^y \equiv \alpha + (\lambda + \beta\mu)(1 - \tau)$  between parents and children. It has a structural component  $\alpha$  reflecting the degree of segregations among the neighborhoods that cannot be lowered with redistribution alone. The other component of intergenerational persistence decreases with the degree  $\tau$  of redistribution and through this channel a policy of redistribution enhances intergenerational social mobility. Next, we characterize the dynamic path of the aggregate state variables.

### 3.2 Dynamics of the Economy-wide State Variables

Given the initial lognormal distribution, by (23) and (24), physical and human capital and income remain lognormally distributed over time such that at each date  $t$ ,  $M_t$  satisfies

$$(26) \quad m_{kt+1} = \ln \bar{s}_2 + (1 - \lambda - \mu) \ln l + \lambda m_{kt} + \mu m_{ht} \\ + \tau (2 - \tau) (\lambda^2 \Delta_{kt}^2 + \mu^2 \Delta_{ht}^2 + 2\lambda\mu cov_t) / 2,$$

$$(27) \quad \Delta_{kt+1}^2 = (1 - \tau)^2 (\lambda^2 \Delta_{kt}^2 + \mu^2 \Delta_{ht}^2 + 2\lambda\mu cov_t),$$

$$(28) \quad m_{ht+1} = \ln \kappa + \varphi + \beta \ln \bar{s}_1 + \beta(1 - \lambda - \mu) \ln l + \beta \lambda m_{kt} \\ + (\alpha + \beta\mu) m_{ht} + \beta\tau (2 - \tau) (\lambda^2 \Delta_{kt}^2 + \mu^2 \Delta_{ht}^2 + 2\lambda\mu cov_t) / 2,$$

$$(29) \quad \Delta_{ht+1}^2 = \sigma^2 + \beta^2 \lambda^2 (1 - \tau)^2 \Delta_{kt}^2 + (\alpha + \beta\mu (1 - \tau))^2 \Delta_{ht}^2 \\ + 2\beta\lambda (1 - \tau) (\alpha + \beta\mu (1 - \tau)) cov_t,$$

$$(30) \quad cov_{t+1} = \beta\lambda^2 (1 - \tau)^2 \Delta_{kt}^2 + \mu (1 - \tau) (\alpha + \beta\mu (1 - \tau)) \Delta_{ht}^2 \\ + \lambda (1 - \tau) (\alpha + 2\beta\mu (1 - \tau)) cov_t.$$

In line with Benabou (2002) we define for each date  $t$  an index of inequality  $\Lambda_t$  as the logarithm of the ratio of mean to median income.

Lemma 4: *At each date  $t$ , inequality index  $\Lambda_t$  equals the variance of logarithmic earnings of agents such that  $\Lambda_t = (\lambda^2 \Delta_{kt}^2 + \mu^2 \Delta_{ht}^2 + 2\lambda\mu cov_t) / 2$  and the evolution of earnings of adults is governed by a lognormal distribution such that*

*$\ln y_t^i \sim N(\lambda m_{kt} + \mu m_{ht} + (1 - \lambda - \mu) \ln l, 2\Lambda_t)$ . The break-even level of income  $\tilde{y}_t$  at which an agent's net tax obligation is zero satisfies:*

$$(31) \quad \ln \tilde{y}_t = \ln y_t + (1 - \tau) \Lambda_t \\ = \lambda m_{kt} + \mu m_{ht} + (1 - \lambda - \mu) \ln l + (2 - \tau) \Lambda_t.$$

The following Lemma describes the equilibrium dynamics of per capita income and inequality jointly.

Lemma 5: *At equilibrium, the time series of the per capita income and income inequality satisfy*

(32)

$$\begin{aligned} \ln y_{t+1} - \ln y_t = & \psi + \mu\varphi + (1 - \alpha)(1 - \lambda - \mu) \ln l - (1 - \alpha - \lambda - \beta\mu) \ln y_t \\ & - \alpha\lambda \ln y_{t-1} + \Lambda_{t+1} - (\alpha + (\lambda + \beta\mu)(1 - \tau)^2) \Lambda_t + \alpha\lambda(1 - \tau)^2 \Lambda_{t-1}, \end{aligned}$$

$$\text{where } \Lambda_{t+1} = \left( \begin{array}{l} \mu^2\sigma^2 + (\lambda + \mu\beta)^2(1 - \tau)^2\lambda^2\Delta_{kt}^2 + (\alpha + (\lambda + \beta\mu)(1 - \tau))^2\mu^2\Delta_{ht}^2 \\ + 2\lambda\mu(1 - \tau)(\alpha(\lambda + \beta\mu) + \beta\mu(\beta\mu + 2\lambda)(1 - \tau))\text{cov}_t \end{array} \right) / 2.$$

## 4 Steady State and Redistribution

The economy converges to a unique steady state.

When there is no endogenous growth, state variables  $(m_{kt}, m_{ht}, \Delta_{kt}^2, \Delta_{ht}^2, \text{cov}_t)$  will converge to its steady states such that  $m_{kt} = m_k$ ,  $\Delta_{kt}^2 = \Delta_k^2$ ,  $m_{ht} = m_h$ ,  $\Delta_{ht}^2 = \Delta_h^2$ , and  $\text{cov}_t = \text{cov}$ . The following Proposition gives a sufficient condition for the existence of a unique steady state to which the equilibrium sequence of  $M_t$  converges.

PROPOSITION 1: *If  $(1 - \alpha)(1 - \lambda) - \beta\mu > 0$  then the equilibrium sequence  $M_t$  monotonically converges to a unique steady state as a function of  $\tau$ .*

*Proof: See Appendix.*

By (27), (29) and (30), in the steady state,  $\Lambda_t = \Lambda$ , where

$$(33) \quad \Lambda \equiv \frac{\mu^2(1 + \lambda\alpha(1 - \tau))}{(1 - \lambda\alpha(1 - \tau))((1 + \lambda\alpha(1 - \tau))^2 - ((\lambda + \beta\mu)(1 - \tau) + \alpha)^2)} \frac{\sigma^2}{2}.$$

By (33),  $\partial\Lambda/\partial\tau < 0$ , that is, an increase in  $\tau$  decreases the steady state inequality  $\Lambda$ .

By (11) the labour supply is a time-invariant function of  $\tau$ . The term  $\psi$  is a constant if both subsidies are applied. In general, it is a function of  $\tau$ . In the steady state, by (32), per

capita income satisfies

$$(34) \quad \ln y = \frac{\psi + \mu\varphi + (1 - \alpha)(1 - \lambda - \mu) \ln l + (1 - \alpha)\Lambda - \frac{((1 - \alpha)\lambda + \beta\mu)(p^y - \alpha)^2 \Lambda}{(\lambda + \beta\mu)^2}}{(1 - \alpha)(1 - \lambda) - \mu\beta}.$$

From Section 3.1, we note that as  $\tau$  increases  $p^y$  decreases and by (34), this decrease contributes to the increase of the steady state income per capita. Thus, an increase in the intergenerational mobility delivers a dynamic gain from redistribution. In addition, by (33) redistribution helps to reduce the static inequality  $\Lambda$ .

### Macro Gains Versus Micro Losses from Redistribution.

Equation (25) shows the dynamic gains from promoting the intergenerational social mobility and equation (34) gives a further proof to show how greater mobility due to greater redistribution helps to increase output per capita.

In this subsection, we provide further explanation of the static gains from redistribution. Let us define broadly a notion of agent  $i$ 's composite capital  $K^i \equiv (k^i)^{\lambda/(1-\varepsilon)} (h^i)^{\mu/(1-\varepsilon)}$ , which, by Lemmas 2–3 and (2), has a positive correlation with her earning,  $y^i = (K^i)^{1-\varepsilon} l^\varepsilon$ , and has a negative correlation with her productivity,

$$(35) \quad MPK^i = (1 - \varepsilon) (K^i)^{-\varepsilon} (l)^\varepsilon.$$

It implies that the poor with low  $K^i$  have higher productivity than the rich.

Consequently, to lower the extent of interpersonal variations in the holdings of composite capital would facilitate mobilization of resources from a low to high end of productivity and then raise national output. The following Lemma shows explicitly the channel through which a greater degree of redistribution leads to a lower variation of productivity. It also provides further explanation as to why inequality  $\Lambda$  and output per capita are negatively related.

*Lemma 6: A permanent increase in  $\tau$  implies a decrease in the steady state variance of  $\ln MPK^i$ , where  $\text{var}(\ln MPK^i) = 2 \left(\frac{\varepsilon}{1-\varepsilon}\right)^2 \Lambda$ .*

*Proof: See Appendix.*

Lemma 6 clearly shows that level of inequality  $\Lambda$  directly affects growth-promoting potential of a redistributive policy. The costs of redistribution such as the negative effects of tax on labour supply, saving and investment, summarized by Lemmas 2 and 3, arise only at the

individual micro level.<sup>4</sup> When the macro gains exceed the micro losses from redistribution, optimal redistributive income tax rate could be positive.

## 5 A Closed-Form Expression for TFP

The conventional measure of TFP for the economy is given by the ratio of the average output to the weighted average of inputs. We use a Cobb-Douglas production technology similar to those assigned to each individual to compute the TFP for the economy as follows

$$(36) \quad TFP \equiv \frac{\int_0^1 y^i di}{\left(\int_0^1 k^i di\right)^\lambda \left(\int_0^1 h^i di\right)^\mu \left(\int_0^1 l^i di\right)^{1-\lambda-\mu}}.$$

Most empirical studies use the above definition of TFP. We now characterize the equilibrium restriction of the model that the above definition of TFP satisfies.

### 5.1 Derivation of TFP

In the equilibrium and at steady state, by (36) and (2), we have the following Proposition.

PROPOSITION 2: *Total factor productivity satisfies*

$$(37) \quad TFP = \exp \left( \frac{\left( \begin{array}{c} ((\lambda - 1)(1 - \alpha\lambda(1 - \tau)) + 2\lambda\beta\mu(1 - \tau)^2) \lambda \Delta_k^2 \\ - (1 - \mu - \alpha\lambda(1 + \mu)(1 - \tau) - 2\lambda\mu\beta(1 - \tau)^2) \mu \Delta_h^2 \end{array} \right)}{2(1 - \lambda(1 - \tau)(\alpha + 2\beta\mu(1 - \tau)))} \right)$$

$$= g(\alpha, \beta, \lambda, \mu, \sigma^2, \tau).$$

where

$$(38) \quad \Delta_k^2 = \frac{\mu^2(1 - \tau)^2(1 + \lambda\alpha(1 - \tau))}{(1 - \lambda\alpha(1 - \tau))((1 + \lambda\alpha(1 - \tau))^2 - ((\lambda + \beta\mu)(1 - \tau) + \alpha)^2)} \sigma^2$$

---

<sup>4</sup>To describe the negative effects of redistribution on the two forms of investments when we remove investment subsidies we replace  $\bar{s}_1$  and  $\bar{s}_2$  by  $s_1(\tau)$  and  $s_2(\tau)$  given by (12) and (13) respectively in the expression for  $\ln \varpi$  above.



$$(39) \quad \Delta_h^2 = \frac{(1 - \lambda(1 - \tau)(\alpha + 2\beta\mu(1 - \tau)))(1 - \lambda^2(1 - \tau)^2) - 2\mu\beta\lambda^3(1 - \tau)^4}{(1 - \lambda\alpha(1 - \tau))((1 + \lambda\alpha(1 - \tau))^2 - ((\lambda + \beta\mu)(1 - \tau) + \alpha)^2)} \sigma^2$$

By (27), (29) and (30), we know in the steady state, the variances of the logarithm of physical and human capital,  $\Delta_k^2$  and  $\Delta_h^2$ , are determined by institutional parameters, such as the degree of social segregation  $\alpha$ , quality of education system  $\beta$  and share of physical  $\lambda$  and human  $\mu$  capital, demographic parameter  $\sigma^2$  and fiscal policy parameter  $\tau$ . It then, follows from the above equation that TFP is determined by the parameters  $\alpha, \beta, \lambda, \mu, \sigma^2$  and  $\tau$ . The above equation provides an explicit formula for TFP which we exploit to generate implications for TFP variations across countries with different characteristics as measured by variations of the parameters  $\alpha, \beta, \lambda, \mu, \sigma^2$  and  $\tau$ . In the case of Benabou (2002), where  $\lambda = 0$ , the TFP turns out to be negatively related to income inequality as described by the following Lemma.

Lemma 7: *When  $\lambda = 0$ , (37) becomes*

$$(40) \quad A_{BU} = \exp\left(\frac{(\mu - 1)\Lambda}{\mu}\right).$$

We know  $\mu - 1 < 0$ . The above formula shows that inequality is negatively related to TFP. It implies that in the economy with a higher level of income inequality, total factor productivity would be lower. This leads to a clear policy implication that reducing inequality is not only good for building a stable social environment from a political point of view, but also helps to improve the overall productivity. Redistribution which lowers inequality helps to improve TFP indirectly.

## 6 Possible Sources of TFP Variations Across Countries

By equation (37), we can identify that institutional factors, such as the degree of social segregation  $\alpha$ , quality of education system  $\beta$ , characteristics of technology described by  $\lambda$  and  $\mu$ , which denote the shares of physical and human capital respectively, and the degree  $\tau$  of redistribution that affect TFP in the model. By (37) in the absence of human capital externality, the rate of time preference and hence the discount factor  $\rho$  does not affect TFP but, by (12) and (13), affects accumulation of physical and human capital. In this section,

we plot graphs of TFP against parameters  $\alpha$ ,  $\beta$ ,  $\lambda$ ,  $\mu$  and  $\tau$  using the baseline values for other parameters.

Property 1: *A more segregated society would have a lower level of TFP .*

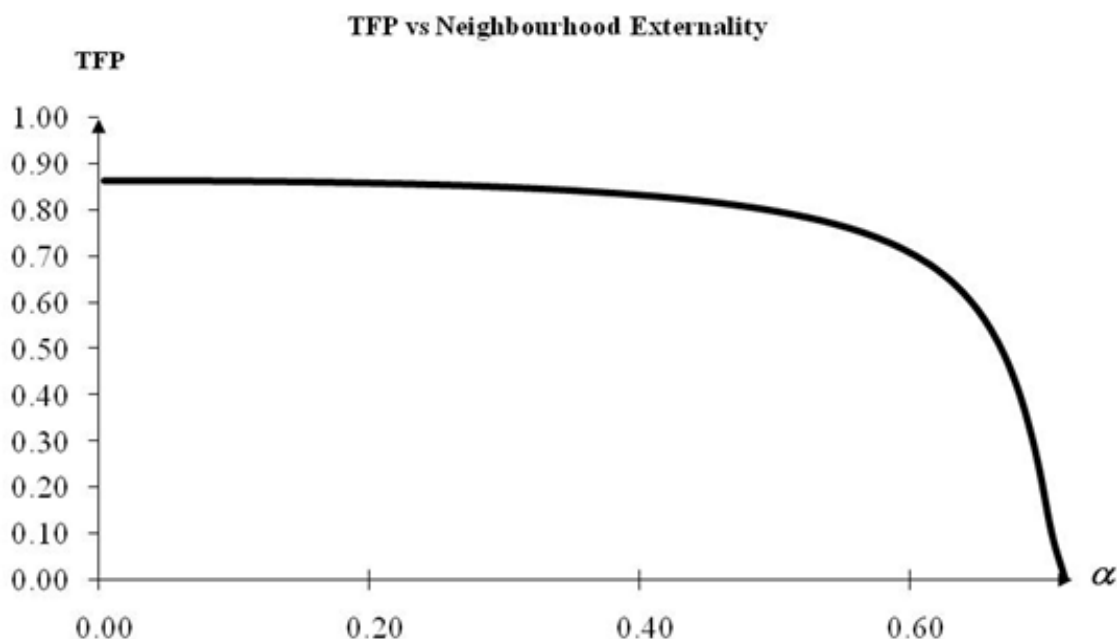


Figure 2—TFP vs Neighbourhood Externality, where  $\lambda = 0.3$ ,  $\mu = 0.5$ ,  $\varepsilon = 0.2$ ,  $\beta = 0.4$ ,  $\sigma^2 = 1$ ,  $\tau = 0$ .

In our model, we interpret  $\alpha$  in the human capital accumulation equation (4) as the parameter that captures the economy's degree of segregation generating a "neighborhood-externality". Segregation of different kinds prevent interaction across communities and enhances the value of family connection in building one's human capital. Englander and Mittelstadt (1988) find a similar conclusion. Segregation prohibits spillover of knowledge across families and communities and then hinders the improvement of overall productivity. The examples of segregation are like interpersonal differences in productivity due to barriers to education, communication problems due to language difficulties, or discrimination because of race, religions, gender, national identity, and age. Because such segregation helps to confine knowledge within families and creates knowledge-gaps across communities, they

cause a persistent difference in marginal productivity across individuals. These interpersonal differences in productivity due to barriers to knowledge spillover implies a lower level of TFP.

Property 2: *A better quality of educational system imply a lower level of TFP.*

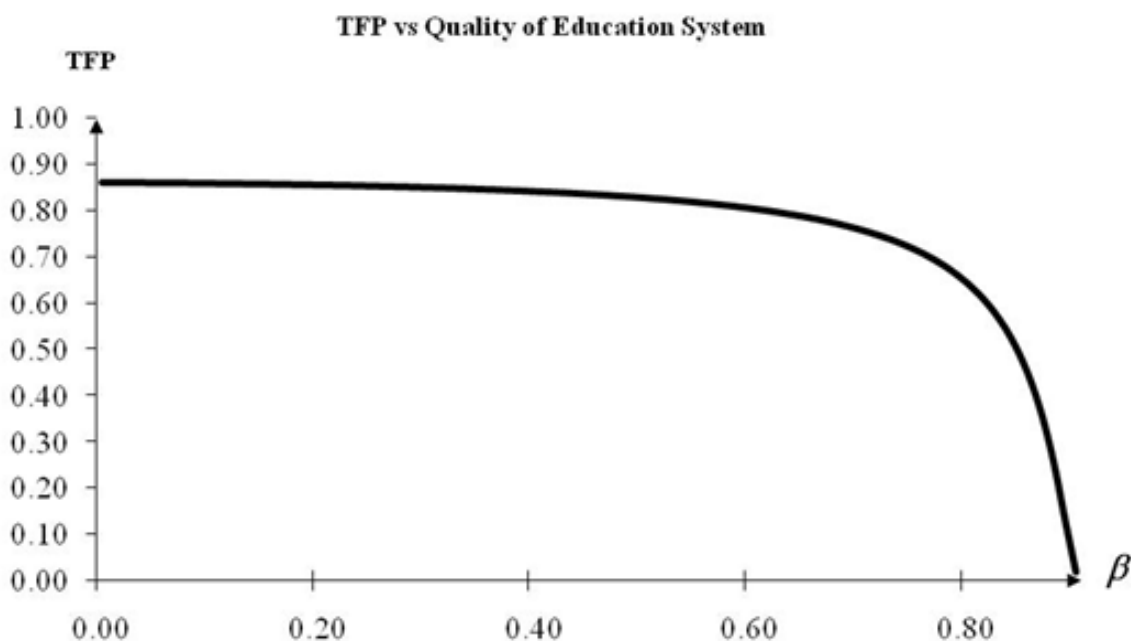


Figure 2—TFP vs Quality of Education System, where  $\lambda = 0.3$ ,  $\mu = 0.5$ ,  $\varepsilon = 0.2$ ,  $\alpha = 0.35$ ,  $\sigma^2 = 1$ ,  $\tau = 0$ .

Figure 2 shows that TFP decreases with the education quality parameter  $\beta$ . By (12) and (13) the optimal proportion of income that agents invest in schooling for their children increases with this parameter and that leads to a higher degree of inequality. A greater inequality, by Lemma 6, increases the variance of productivity in the economy and that imply a lower level of TFP.

Property 3: *In presence of capital-skill complementarity, a more capital intensive technology implies a profile of TFP that has a U shape.*

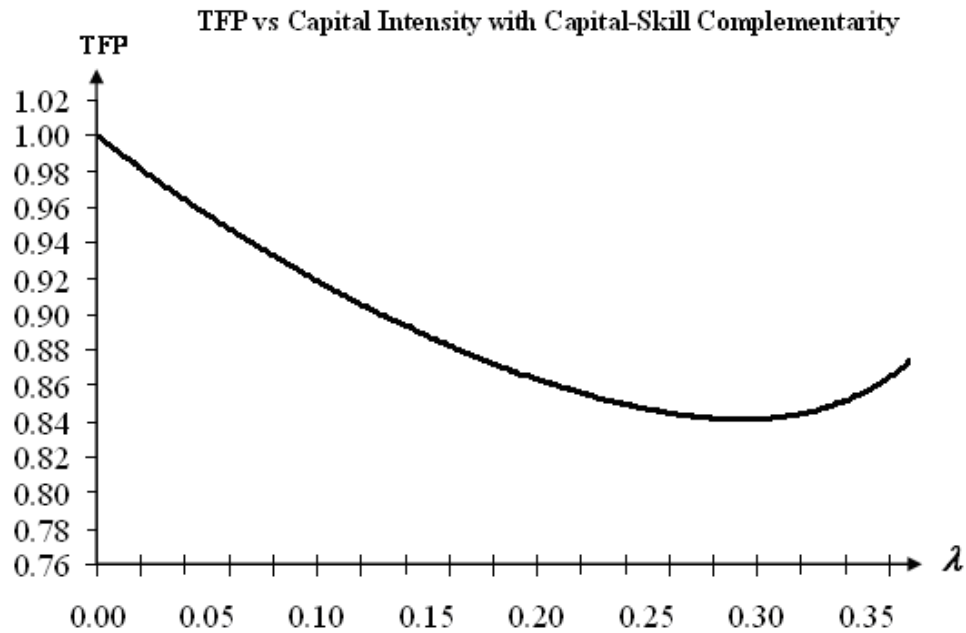


Figure 3—TFP vs Capital Intensity with Capital-Skill Complementarity, where  $\alpha = 0.35$ ,  $\beta = 0.4$ ,  $\sigma^2 = 1$ ,  $\tau = 0$ .

Figure 3 illustrates differences among economies that arise from choice of technology and stages of development. Economies in their earlier stages of development typically rely on technology with a large intensity of unskilled labor. In the later stages of development it adopts technology that relies more intensely on machines and tools and requires skilled labor to operate those machines due to a well-known hypothesis of "capital-skill complementarity". In the process, unskilled labor becomes less important and gets replaced by modern machines. To capture this hypothesized path of development, in our simulations, we decrease  $\varepsilon$  and increase  $\lambda$  and  $\mu$  proportionately by keeping the ratio of the two capital shares same as implied by the Barro, Mankiw and Xala-I-Martin (1995) estimates. The TFP follows a U-shaped pattern as the technology replaces unskilled labor by machines accompanied by their skilled operators. As the relative value of human and physical capital to raw labor increases, the value of the privileged owners of capital relative to raw labor increases. The increased income inequality and associated increase in interpersonal productivity variance diminishes TFP in the economy. Note from Figure 3 that TFP increases eventually as the the elasticity of unskilled labor falls below a critical value. This feature illustrates an

implication of *Lemma 6* that variations of productivity decreases and hence TFP increases for a sufficiently low value of unskilled labor in technology.

Property 4: *A relatively more human capital intensive than physical capital intensive technology implies an inverted U shaped TFP profile.*

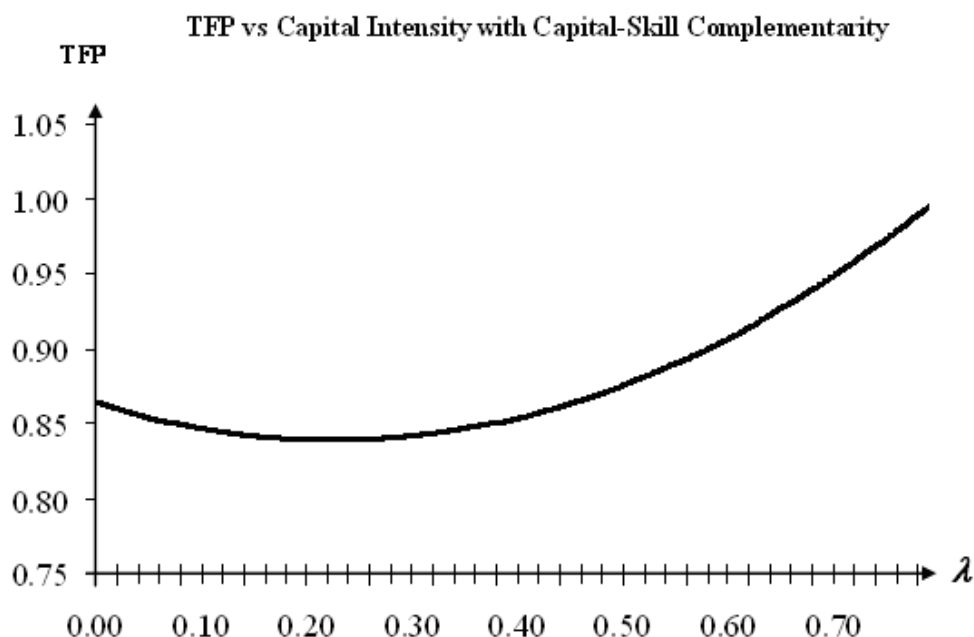


Figure 4—TFP vs Capital Intensity, where  $\alpha = 0.35$ ,  $\beta = 0.4$ ,  $\lambda + \mu = 0.8$ ,  $\sigma^2 = 1$ ,  $\tau = 0$ .

Figure 4 illustrates an implicit property of capital skill complementarity that the TFP reaches its minimum when the technology values both types of types of capital symmetrically by setting  $\lambda = \beta\mu$ , which corresponds to the minimum variance of productivity across individuals. Note also that harmful effects of inequality arise primarily from the difference in the marginal product of human capital due to the complementarity between the child's innate ability shock and parental human capital. As the relative importance of human capital diminishes the growth augmenting "investment reallocation effect" from reduced inequality also diminishes. TFP increases from its minimum as the technology assigns very different values to human and physical capital since it increases interpersonal variations in productivity.

Property 5: *An economy with a higher degree of redistribution would have a higher level of TFP.*

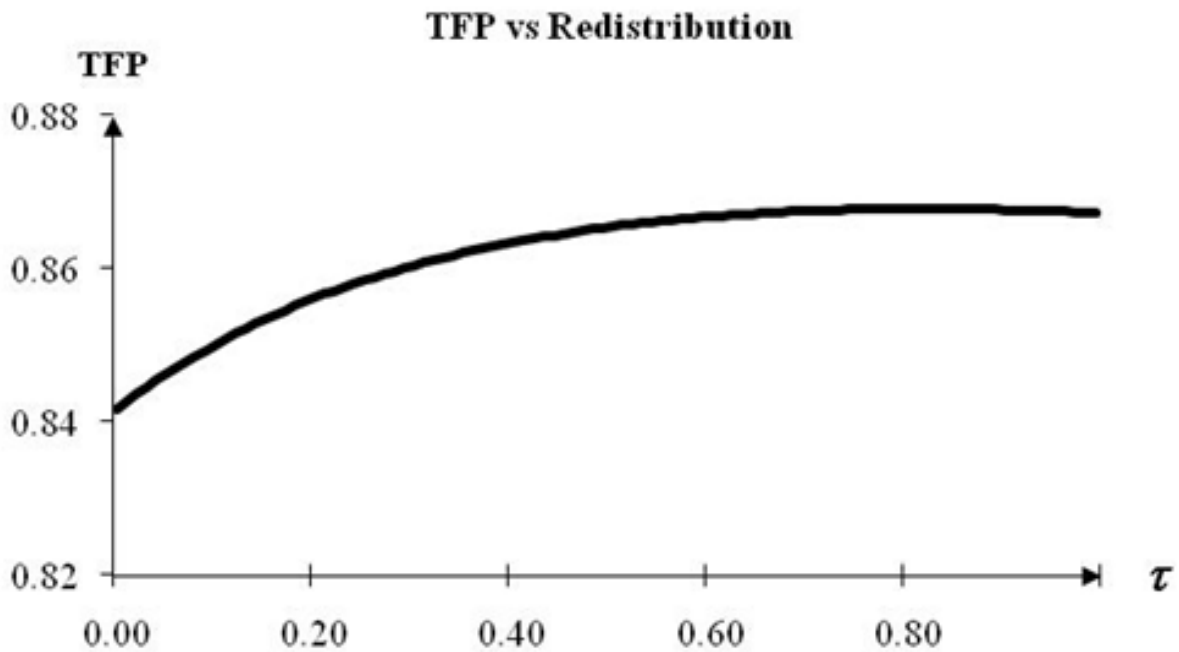


Figure 5—TFP vs Redistribution, where  $\lambda = 0.3$ ,  $\mu = 0.5$ ,  $\varepsilon = 0.2$ ,  $\alpha = 0.35$ ,  $\beta = 0.4$ ,  $\sigma^2 = 1$ .

Figure 5 shows that the level of TFP increases with higher degrees of redistribution. This implies that in an economy without a credit market, the redistribution of income plays an important role as lubricant in helping the economy to improve its total factor productivity. In the absence of a credit market, the whole economy's productivity would be improved if the government could redistribute income from the rich to the poor to help those children who were born in poor families and have no money to afford schooling. After redistribution, poor bright children could go to school and have a higher level of education and thus produce greater output in the future. Hence, we can say that higher degrees of redistribution further reduce the imbalance of opportunities of taking education because of credit constraints. TFP will be improved accordingly. It decreases a little when  $\tau$  is very high.

By (26) and (28), we identify those institutional and policy parameters also affect inputs. In order to estimate how much TFP helps to explain the differences in per capita income across countries, we also need to check differences in inputs. Therefore, in the following section, we compare differences of inputs and differences of TFP in explaining differences

on the per capita income differences<sup>5</sup>.

## 7 Effects of TFP Differences on the Per Capita Income Differences

In this section, we discuss the extent to which TFP differences help to explain per capita income differences across countries. First, we provide the elasticity of TFP with respect to institutional and policy parameters and compare them. Second, we discuss how much the differences of per capita income could be explained by TFP and other inputs. We change the value of each parameter by 1% while keeping the values of all other parameters constant at their baseline values and note the percentage changes in TFP to calculate the elasticities of TFP with respect to its determinants and summarize them in Table 1.

Table 1: Elasticity of TFP with respect to  $\alpha$ ,  $\beta$ ,  $\lambda$ ,  $\mu$  and  $\tau$

$\alpha, \beta, \lambda, \mu$ and $\tau$	$\alpha$	$\beta$	$\lambda/\mu = 3/5$	$\lambda + \mu = 0.8$	$\tau$
0.1	-0.1	-0.2	-8.5	-2.2	1.0
0.2	-0.5	-0.4	-6.2	-0.9	0.7
0.3	<b>-1.1</b>	-0.7	<b>-2.5</b>	<b>0.3</b>	<b>0.5</b>
0.4	-4.0	<b>-1.0</b>	–	1.5	0.4
0.5	-4.7	-1.7	–	2.6	0.2
0.6	-13.9	-3.0	–	3.6	0.2

The first column lists various points where we calculate the elasticities. The second through sixth columns show the elasticities of TFP with respect to those parameters respectively and corresponding to those values ranging from 0.1 to 0.6. For example, as the degree of segregation  $\alpha$  increases by 1% from 0.1, the logarithm of TFP decreases by 0.1%. As  $\alpha$  increases up to 0.6, the elasticity of TFP decreases to -13.9%. This implies that as  $\alpha$  increases, TFP decreases at an increasing rate. This finding is consistent with Figure 1. If we compare elasticities of TFP with respect to institutional parameters with the elasticity of TFP with respect to the redistributive policy parameter, we can see that the former are much greater than those of the latter. This implies that TFP is more sensitive to variations in the institutional structure than redistributive policy.

<sup>5</sup>Note we allow for both education and bequest subsidies in the following simulation to offset the distortionary effects of income taxes on investment in schooling and bequest to determine the maximum possible beneficial effects of a redistributive policy on TFP.

In order to answer how much differences of per capita income across countries could be explained by differences in inputs and differences in TFP, taking the logarithm of (A.55) and calculating the first difference, we have

$$(41) \quad \Delta \ln y = \Delta \ln TFP + \lambda \Delta \ln k + \mu \Delta \ln h + (1 - \lambda - \mu) \Delta \ln l.$$

From equation (41), we can see that the percentage of variations of per capita income explained by differences of TFP is  $\Delta \ln TFP / \Delta \ln y$  while differences of inputs explain by  $1 - \Delta \ln TFP / \Delta \ln y$ .

Using equation (41), we decompose total variation of per capita income due to variations in various parameters discussed above into variations of TFP and inputs to determine how much of the difference of per capita income could be explained by differences in inputs and differences in TFP. Note that by (41) the last four columns must sum to 100%.

Table 2: Variations of Per Capita Income Explained by TFP and Inputs  
[Change: vary  $\alpha$  (degree of segregation) by 1 % at different points]

$\alpha$	$\Delta y\%$	TFP	physical capital	human capital	labor
0.1	-5.58%	2.26%	30.00%	68.00%	-0.25%
0.2	-7.52%	6.89%	30.00%	63.31%	-0.20%
0.3	-10.93%	<b>10.10%</b>	30.00%	60.05%	-0.15%
0.4	-17.66%	12.26%	30.00%	57.85%	-0.11%
0.5	-34.16%	13.62%	30.00%	56.44%	-0.06%
0.6	-96.81%	14.39%	30.00%	55.63%	-0.02%

The above Table shows that differences in human capital is the dominant factor explaining the differences of per capita income across countries due to different degrees of segregation. The importance of TFP increases and its differences could explain up to 14.39% of the differences of per capita income. But it is still very small relative to other inputs. Per capita income decreases smoothly with  $\alpha$  when the value of  $\alpha$  is low but it decreases much faster when  $\alpha$  is high. It also implies that without redistribution, and in the absence of credit market, an increase in the degree of segregation drags down per capita significantly, especially when the degree of segregation is already very high.



Table 3: Variations of Per Capita Income Explained by TFP and Inputs

[Change: vary  $\beta$  (quality of educational institution) by 1 % at different points]

$\beta$	$\Delta y\%$	TFP	physical capital	human capital	labor
0.1	-46.68%	0.51%	30.00%	69.68%	-0.19%
0.2	-40.18%	1.01%	30.00%	69.22%	-0.23%
0.3	-44.80%	1.46%	30.00%	68.76%	-0.21%
0.4	-55.85%	<b>1.85%</b>	30.00%	68.33%	-0.18%
0.5	-77.35%	2.18%	30.00%	67.95%	-0.13%
0.6	-121.97%	2.44%	30.00%	67.64%	-0.09%

The second column of the above Table shows the elasticity of per capita income with respect to  $\beta$ , i.e., the percentage change of per capita income due to 1% increase of  $\beta$ . What the results show is that per capita income is very sensitive to the quality of the education system. The third through sixth columns provide how much of the difference of per capita income explained by differences in TFP and differences in other inputs. From Table 2, we can see that if countries are different due to the quality of the education system, most of the differences in per capita income are explained by differences in human and physical capital. Differences of physical capital always explain 30% of the differences of per capita income because changes in  $\beta$  do not have any effect on the ratio of physical capital to per capita income. But differences in TFP has a negligible effect relative to inputs.

Table 4: Variations of Per Capita Income Explained by TFP and Inputs

[Change: vary  $\lambda$  (share of physical capital) by 1 % at different points keeping  $\lambda/\mu = 3/5$ ]

$\lambda$	$\Delta y\%$	TFP	physical capital	human capital	labor
0.10	-57.92%	14.71%	71.26%	22.88%	-8.85%
0.20	-43.64%	14.26%	82.70%	17.25%	-14.21%
0.30	-50.39%	<b>4.91%</b>	82.69%	28.86%	-16.46%

If countries are different due to different quantities of physical  $\lambda$  and human  $\mu$  capital, then physical and human capital accumulation dominate the differences of per capita income. The role of TFP becomes less important as  $\lambda$  and  $\mu$  increase. This implies that as countries develop from an early stage to an advanced stage without changing other institutional parameters, TFP contributes less to the differences in per capita income.

Table 5: Variations of Per Capita Income Explained by TFP and Inputs

[Change: vary  $\lambda$  (share of physical capital) by 1 % at different points keeping labour share  $\varepsilon = 0.2$ ]

$\lambda$	$\Delta y\%$	TFP	physical capital	human capital	labor
0.1	-93.42%	2.31%	38.83%	58.95%	-0.09%
0.2	-73.74%	1.18%	50.41%	48.53%	-0.12%
0.3	-70.11%	<b>-0.05%</b>	65.16%	35.45%	-0.13%
0.4	-69.90%	-2.10%	80.58%	21.66%	-0.13%
0.5	-71.28%	-3.59%	96.23%	7.49%	-0.13%
0.6	-72.88%	-4.95%	112.41%	-7.33%	-0.13%

If the sum of  $\lambda$  and  $\mu$  is constant, i.e., the share of unskilled labour does not change, the effect of differences in TFP on the variation of cross-country income becomes much smaller. This means that if a country starts to rely more on physical capital and less on skilled labour, the role of human capital and TFP becomes less important while the role of physical capital becomes more and more important in explaining differences in per capita income. It also implies that human capital and TFP are closely related to each other.

Table 6: Variations of Per Capita Income Explained by TFP and Inputs

[Change: vary  $\tau$  (degree of redistribution) by 1 % at different points]

$\tau$	$\Delta y\%$	TFP	physical capital	human capital	labor
0.1	2.99%	34.00%	30.00%	50.87%	-14.87%
0.2	1.34%	52.97%	30.00%	53.31%	-36.28%
0.3	0.15%	<b>334.25%</b>	30.00%	96.40%	-360.65%
0.4	-0.78%	-44.56%	30.00%	36.83%	77.73%
0.5	-1.58%	-14.92%	30.00%	40.55%	44.37%
0.6	-2.36%	-6.35%	30.00%	40.79%	35.57%

By way of contrast, Table 6 shows that if countries have different fiscal policies but similar institutional parameters, differences in TFP plays a very important role in explaining the differences in per capita income. For example, as the degree of redistribution  $\tau$  increases from 0.2 to 0.3, differences in TFP explains more than one hundred percent in explaining the differences of per capita income. This is because that as  $\tau$  increases from 0.2 to 0.3, physical and human capital increase but the labour input decreases a lot. Therefore, the ratio of the increase of TFP to the increase of income is more than one, i.e., differences of per capita income explained by TFP differences is more than one hundred percent. As  $\tau$  is greater than

30%, which is greater than the optimal degree of redistribution, per capita income will be lower than its maximum and the change of income becomes negative. Therefore, the ratio of the negative change of per capita income to the positive change of TFP is negative. That is why we get negative percentages in Table 6. It also means that higher TFP does not mean higher income, because of lower levels of inputs which drag down the aggregate income.

Table 7 summarizes the role of TFP in explaining the difference in per capita income from the above Tables as countries differ due to different  $\beta$ ,  $\alpha$ ,  $\lambda$ ,  $\mu$  and  $\tau$ .

Table 7: Variations of Per Capita Income Explained by TFP

[Change: vary  $\alpha$ ,  $\beta$ ,  $\lambda$ ,  $\tau$  by 1 % at different points]

$\alpha, \beta, \lambda, \tau$	$\alpha$	$\beta$	$\lambda/\mu = 3/5$	$\lambda + \mu = 0.8$	$\tau$
0.1	2.26%	0.51%	14.71%	2.31%	-34.00% (-)
0.2	6.89%	1.01%	14.26%	1.18%	-52.97% (-)
0.3	<b>10.10%</b>	1.46%	<b>4.91%</b>	<b>-0.05%</b>	<b>-334.25% (-)</b>
0.4	12.26%	<b>1.85%</b>	–	-2.10%	-44.56% (-)
0.5	13.62%	2.18%	–	-3.59%	-14.92% (-)
0.6	14.39%	2.44%	–	-4.95%	-6.35% (-)

Table 7 shows that differences in per capita income explained by differences in TFP is quite small for economies where differences of TFP arise solely from different institutional parameters,  $\beta$  and  $\alpha$ , or technology parameters,  $\lambda$  and  $\mu$ . For example, the maximum of the differences of income explained by differences in TFP is 2.44% or 14.39% when one country's  $\beta$  or  $\alpha$  is equal to 0.55 while another country's  $\beta$  or  $\alpha$  is equal to 0.60. When  $\lambda$  and  $\mu$  are fixed at a ratio of 3/5, differences in TFP could help to explain differences of per capita income up to 14.71%. Keeping the share of labour at 0.2 but changing  $\lambda$  and  $\mu$ , the difference of income explained by differences in TFP is even lower. This implies that differences in inputs are the driving force in explaining the difference of per capita income across countries when countries differ in their institutional parameters. But, from the last column, we can see that differences in TFP play an important role in explaining the difference of per capita income across countries when differences in TFP arise from differences in fiscal policies. It simply means that, according to the model, economic policy may improve productivity by reducing communication barriers and income inequality but those policies also lower inputs in to production. Therefore, the aggregate income may not increase.

The above discussion help us to understand not only how and why TFP varies across countries but also to understand when the variation of per capita income across countries

could be largely explained by differences in TFP rather than by differences in accumulation of inputs.

## **8 Concluding Remarks**

We present an analytical expression for TFP based on a dynamic general equilibrium theory to estimate quantitatively how much of the variations of per capita income across countries can be explained by variations in TFP as opposed to variations in other inputs. Our theory of TFP indicates that measured TFP would be lower in a country with a greater degree of segregation that hinders knowledge spillover, and with a better quality of educational system. It would also be lower in a country which operates a technology with a smaller elasticity of unskilled labour, provided it is not too small, and with a relatively symmetric distribution of elasticity of physical and human capital. A country with a higher degree of redistribution would have a higher level of TFP.

From the numerical simulations of the above model, we find that TFP differences do not explain much of the differences of per capita income if the sources of variations of TFP consist of institutional and technological differences. TFP differences, however, play a significant role in explaining the differences of per capita income if the sources of variations arrive from variations in the degree of redistributive fiscal policies.

## Appendix

PROOFS OF LEMMAS 1, 2 AND 3:

By (3), (4) and (5), we rewrite (8) as follows:

$$(A.1) \quad \ln U(h_t^i, k_t^i, M_t; T) = \max_{s_{1t}^i, s_{2t}^i, l_t^i} \left\{ (1 - \rho) [\ln(1 - s_{1t}^i - s_{2t}^i) - \ln(1 + \theta) + \ln \tilde{y}_t^i - (l_t^i)^\eta] + \rho E_t [\ln U(h_{t+1}^i, k_{t+1}^i, M_{t+1}; T)] \right\}.$$

Agent solves (A.1) subject to (2), (6) and

$$(A.2) \quad h_{t+1}^i = \kappa ((1 + d) s_{1t}^i)^\beta \xi_{t+1}^i (k_t^i)^{\beta \lambda (1 - \tau)} (h_t^i)^{\alpha + \beta \mu (1 - \tau)} (l_t^i)^{\beta (1 - \lambda - \mu)(1 - \tau)} (\tilde{y}_t^i)^{\beta \tau}, \text{ and}$$

$$(A.3) \quad k_{t+1}^i = (1 + v) s_{2t}^i (k_t^i)^{\lambda (1 - \tau)} (h_t^i)^{\mu (1 - \tau)} (l_t^i)^{(1 - \lambda - \mu)(1 - \tau)} (\tilde{y}_t^i)^\tau.$$

We guess the value function as:  $\ln U(h_t^i, k_t^i, M_t; T) = Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t$ . Then by substituting this value function into (A.1), we get

$$(A.4) \quad Z_1 \ln h_t^i + Z_2 \ln k_t^i + B_t = (1 - \rho) \left( \begin{aligned} & \ln(1 - s_{1t}^i - s_{2t}^i) / (1 + \theta) \\ & + (1 - \lambda - \mu)(1 - \tau) \ln l_t^i + \tau \ln \tilde{y}_t^i - (l_t^i)^\eta \end{aligned} \right) \\ + (1 - \rho + \rho \beta Z_1 + \rho Z_2) \lambda (1 - \tau) \ln k_t^i \\ + ((1 - \rho + \rho \beta Z_1 + \rho Z_2) \mu (1 - \tau) + \rho \alpha Z_1) \ln h_t^i \\ + \rho \left( \begin{aligned} & Z_1 \left( \begin{aligned} & \ln \kappa + \beta \ln(1 + d) s_{1t}^i + \varphi \\ & + \beta (1 - \lambda - \mu)(1 - \tau) \ln l_t^i + \beta \tau \ln \tilde{y}_t^i \end{aligned} \right) \\ & + Z_2 (\ln(1 + v) s_{2t}^i + (1 - \lambda - \mu)(1 - \tau) \ln l_t^i + \tau \ln \tilde{y}_t^i) + B_{t+1} \end{aligned} \right).$$

Taking partial differentials with respect to  $\ln k_t^i$  and  $\ln h_t^i$  yield

$$(A.5) \quad Z_1 = (1 - \rho + \rho \beta Z_1 + \rho Z_2) \mu (1 - \tau) + \rho \alpha Z_1,$$

$$(A.6) \quad Z_2 = (1 - \rho + \rho \beta Z_1 + \rho Z_2) \lambda (1 - \tau).$$

Rearranging (A.5) and (A.6), we verify the guess and confirm the existence of (A.4) and thus Lemma 1 is established.

The first-order conditions of (A.1) with respect to the saving rates and labour supply are

$$(A.7) \quad \frac{1 - \rho}{1 - s_{1t}^i - s_{2t}^i} = \rho \left( \frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i} \frac{\partial \ln h_{t+1}^i}{\partial s_{1t}^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i} \frac{\partial \ln k_{t+1}^i}{\partial s_{1t}^i} \right),$$

$$(A.8) \quad \frac{1 - \rho}{1 - s_{1t}^i - s_{2t}^i} = \rho \left( \frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i} \frac{\partial \ln h_{t+1}^i}{\partial s_{2t}^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i} \frac{\partial \ln k_{t+1}^i}{\partial s_{2t}^i} \right),$$

$$(A.9) \quad (1 - \rho) \eta (l_t^i)^{\eta-1} = (1 - \rho) (1 - \lambda - \mu) (1 - \tau) / l_{1t}^i \\ + \rho \left( \frac{\partial \ln U_{t+1}^i}{\partial \ln h_{t+1}^i} \frac{\partial \ln h_{t+1}^i}{\partial l_t^i} + \frac{\partial \ln U_{t+1}^i}{\partial \ln k_{t+1}^i} \frac{\partial \ln k_{t+1}^i}{\partial l_t^i} \right),$$

where  $\partial \ln k_{t+1}^i / \partial s_{1t}^i = 0$ ,  $\partial \ln k_{t+1}^i / \partial s_{2t}^i = 1 / s_{2t}^i$ ,  $\partial \ln h_{t+1}^i / \partial s_{1t}^i = \beta / s_{1t}^i$ ,  $\partial \ln h_{t+1}^i / \partial s_{2t}^i = 0$ ,  $\partial \ln k_{t+1}^i / \partial l_{1t}^i = (1 - \lambda - \mu) (1 - \tau) / l_{1t}^i$  and  $\partial \ln h_{t+1}^i / \partial l_{1t}^i = \beta (1 - \lambda - \mu) (1 - \tau) / l_{1t}^i$ .

The above optimization problem (A.4) is strictly concave. Consequently, (A.7)—(A.9) are sufficient for the optimization exercise and the Lemmas 2 and 3 follow immediately after we substitute (9) and (10) into (A.7)—(A.9).  $\square$

PROOF OF LEMMA 4: By assumption, at the initial date  $t = 0$ , physical and human capitals are lognormally distributed. By (23) and (24), it follows that  $k_t^i$  and  $h_t^i$  remain lognormally distributed over time and hence by (2)  $y_t^i$  is lognormal and is given by,

$$(A.14) \quad \ln y_t^i = \lambda \ln k_t^i + \mu \ln h_t^i + (1 - \lambda - \mu) \ln l_t^i.$$

By (11), it follows that the mean of the lognormal distribution of  $y_t^i$  is given by,

$$(A.15) \quad \int_0^1 \ln y_t^i di = \lambda m_{kt} + \mu m_{ht} + (1 - \lambda - \mu) \ln l.$$

The variance of  $\ln y_t^i$  is the sum of variances of  $\ln k_t^i$ ,  $\ln h_t^i$  plus the covariance of these two variables

$$(A.16) \quad \text{var} [\ln y_t^i] = \lambda^2 \Delta_{kt}^2 + \mu^2 \Delta_{ht}^2 + 2\lambda\mu \text{cov}_t.$$

The income per capita  $y_t$ , following Crow and Shimizu's (1988) description about properties

of moment generating function on lognormal distribution on page 9, is

$$(A.17) \quad y_t = \int_0^1 y_t^i di = \exp \left( \int_0^1 \ln y_t^i di + \frac{1}{2} \text{var} [\ln y_t^i] \right).$$

The median income is

$$(A.18) \quad y_{t,median} = \exp \left( \int_0^1 \ln y_t^i di \right).$$

Therefore, inequality index is

$$(A.19) \quad \Lambda_t \equiv \log \left( \frac{y_t}{y_{t,median}} \right) = \frac{1}{2} \text{var} [\ln y_t^i] = (\lambda^2 \Delta_{kt}^2 + \mu^2 \Delta_{ht}^2 + 2\lambda\mu cov_t) / 2.$$

To derive the expression for the break-even point given by (7), we note the mean of  $y_t^i$  in logarithm, by (A.17), satisfies

$$(A.20) \quad \ln y_t = \lambda m_{kt} + \mu m_{ht} + (1 - \lambda - \mu) \ln l + \Lambda_t,$$

and the mean of  $(y_t^i)^{1-\tau}$  in logarithm is

$$(A.21) \quad \ln \int_0^1 (y_t^i)^{1-\tau} di = (1 - \tau) (\lambda m_{kt} + \mu m_{ht} + (1 - \lambda - \mu) \ln l) + (1 - \tau)^2 \Lambda_t.$$

Taking the difference between before and after tax income yields

$$(A.22) \quad \ln y_t - \ln \int_0^1 (y_t^i)^{1-\tau} di = \lambda \tau m_{kt} + \mu \tau m_{ht} + (1 - \lambda - \mu) \tau \ln l + \tau (2 - \tau) \Lambda_t.$$

It means that  $\tau \ln \tilde{y}_t = \lambda \tau m_{kt} + \mu \tau m_{ht} + (1 - \lambda - \mu) \tau \ln l + \tau (2 - \tau) \Lambda_t$ , then we can get (31). This proves Lemma 4.  $\square$

PROOF OF LEMMA 5: Integrating both sides of (25) across all agents  $i$  we get,

$$(A.23) \quad \int_0^1 \ln y_{t+1}^i di = \psi + (1 - \alpha) (1 - \lambda - \mu) \ln l + \mu \varphi + (\lambda + \beta \mu) \tau \ln \tilde{y}_t - \alpha \lambda \tau \ln \tilde{y}_{t-1} \\ + (\alpha + (\lambda + \beta \mu) (1 - \tau)) \int_0^1 \ln y_t^i di - \alpha \lambda (1 - \tau) \int_0^1 \ln y_{t-1}^i di.$$

From (A.17), we know

$$(A.24) \quad \int_0^1 \ln y_t^i di = \ln \int_0^1 y_t^i di - \frac{1}{2} \text{var} [\ln y_t^i].$$

Combining (A.24) with (A.23) yields

$$(A.25) \quad \begin{aligned} \ln \int_0^1 y_{t+1}^i di - \frac{1}{2} \text{var} [\ln y_{t+1}^i] &= \psi + \mu\varphi + (1 - \alpha)(1 - \lambda - \mu) \ln l \\ &\quad + (\lambda + \beta\mu) \tau \ln \tilde{y}_t - \alpha\lambda\tau \ln \tilde{y}_{t-1} \\ &\quad + (\alpha + (\lambda + \beta\mu)(1 - \tau)) \left( \ln \int_0^1 y_t^i di - \frac{1}{2} \text{var} [\ln y_t^i] \right) \\ &\quad - \alpha\lambda(1 - \tau) \left( \ln \int_0^1 y_{t-1}^i di - \frac{1}{2} \text{var} [\ln y_{t-1}^i] \right). \end{aligned}$$

Substituting (31) into (A.25) and by (A.19) yield (32). This proves Lemma 5.  $\square$

PROOF OF LEMMA 6: By (35),

$$(A.26) \quad \begin{aligned} \ln MPK^i &= \ln(1 - \varepsilon) - \varepsilon \ln K^i + \varepsilon \ln l \\ &= \ln(1 - \varepsilon) - \varepsilon \left( \frac{\lambda}{1 - \varepsilon} \ln k^i + \frac{\mu}{1 - \varepsilon} \ln h^i \right) + \varepsilon \ln l. \end{aligned}$$

Therefore, the variance of  $\ln MPK^i$  satisfies,

$$(A.27) \quad \begin{aligned} \text{var} (\ln MPK^i) &= \left( \frac{\lambda\varepsilon}{1 - \varepsilon} \right)^2 \Delta_k^2 + \left( \frac{\mu\varepsilon}{1 - \varepsilon} \right)^2 \Delta_h^2 + 2\lambda\mu \left( \frac{\varepsilon}{1 - \varepsilon} \right)^2 \text{cov} \\ &= \left( \frac{\varepsilon}{1 - \varepsilon} \right)^2 (\lambda^2 \Delta_k^2 + \mu^2 \Delta_h^2 + 2\lambda\mu \text{cov}) \\ &= 2 \left( \frac{\varepsilon}{1 - \varepsilon} \right)^2 \Lambda, \text{ by Lemma 4.} \end{aligned}$$

By (33), a higher value of  $\tau$  corresponds to a lower value of  $\Lambda$  and that proves Lemma 6.  $\square$

PROOF OF PROPOSITION 1: Writing the system of linear equations (26) to (30) in a matrix form, we get

$$(A.40) \quad M_{t+1} = A_0 + A_1 * M_t,$$



where

$$M_{t+1} \equiv \begin{bmatrix} m_{kt+1} \\ m_{ht+1} \\ \Delta_{kt+1}^2 \\ \Delta_{ht+1}^2 \\ \text{COV}_{t+1} \end{bmatrix}, A_0 \equiv \begin{bmatrix} \ln \bar{s}_2 + (1 - \lambda - \mu) \ln l \\ \ln \kappa + \varphi + \beta \ln \bar{s}_1 + \beta(1 - \lambda - \mu) \ln l \\ 0 \\ \sigma^2 \\ 0 \end{bmatrix},$$

$$A_1 \equiv \begin{bmatrix} \lambda & \mu & \tau(2 - \tau)\lambda^2/2 & \tau(2 - \tau)\mu^2/2 & \tau(2 - \tau)\lambda\mu \\ \beta\lambda & \alpha + \beta\mu & \beta\tau(2 - \tau)\lambda^2/2 & \beta\tau(2 - \tau)\mu^2/2 & \beta\tau(2 - \tau)\lambda\mu \\ 0 & 0 & (1 - \tau)^2\lambda^2 & (1 - \tau)^2\mu^2 & 2\lambda\mu(1 - \tau)^2 \\ 0 & 0 & (1 - \tau)^2\beta^2\lambda^2 & (\alpha + \beta\mu(1 - \tau))^2 & 2\beta\lambda(1 - \tau)(\alpha + \beta\mu(1 - \tau)) \\ 0 & 0 & (1 - \tau)^2\beta\lambda^2 & \mu(1 - \tau)(\alpha + \beta\mu(1 - \tau)) & \lambda(1 - \tau)(\alpha + 2\beta\mu(1 - \tau)) \end{bmatrix}.$$

The sequence  $M_t$  converges to a steady state if all eigenvalues of  $A_1$ , denoted by  $S(A_1) \equiv (E_j)$ ,  $j = 1, 2, 3, 4, 5$ , are less than one<sup>6</sup>. By setting  $\tau = 0$  to avoid unnecessary details, we solve  $\det |A_1 - S(A_1)I| = 0$ , where  $I$  is identity matrix, to get

(A.41)

$$E_1 \equiv \lambda\alpha, E_2 \equiv \left( \beta\mu + \lambda + \alpha + \sqrt{(\beta^2\mu^2 + 2\beta\lambda\mu + 2\alpha\beta\mu + \lambda^2 - 2\lambda\alpha + \alpha^2)} \right) / 2,$$

$$E_3 \equiv \left( \beta\mu + \lambda + \alpha - \sqrt{(\beta^2\mu^2 + 2\beta\lambda\mu + 2\alpha\beta\mu + \lambda^2 - 2\lambda\alpha + \alpha^2)} \right) / 2.$$

Eigenvalues  $E_1$ ,  $E_2$  and  $E_3$  are less than one since  $\alpha, \beta, \lambda, \mu \in (0, 1)$ ,  $\lambda + \mu < 1$  and  $(1 - \alpha)(1 - \lambda) - \beta\mu > 0$ . Note that when  $\tau = 0$ ,  $|A_1| = \lambda^4\alpha^4$ . It implies that  $E_1 * E_2 * E_3 * E_4 * E_5 = \lambda^4\alpha^4$ , since  $A_1$  is symmetric. By (A.41),  $E_1 * E_2 * E_3 = \lambda^2\alpha^2$ . It follows, therefore,

$$(A.42) \quad E_4 E_5 = \lambda^2 \alpha^2.$$

The symmetry of  $A_1$  implies also that the *trace*  $\{A_1\} = \sum_{j=1, \dots, 5} E_j$ . Or, equivalently,

$$(A.43) \quad E_4 + E_5 = \lambda(\lambda + 2\beta\mu) + (\alpha + \beta\mu)^2.$$

<sup>6</sup>For detailed discussion about this property, please see Reich (1949), Lorenz (1993) and Young (2003).

By (A.42) and (A.43) and the assumption  $(1 - \alpha)(1 - \lambda) - \beta\mu > 0$ , it follows that both  $E_4 < 1$  and  $E_5 < 1$ . Thus,  $S(A_1) < 1$ . Consequently,  $M_t$  converges to a unique steady state which we denote as  $M$ . By (A.40),  $M$  satisfies the following fixed point problem

$$(A.44) \quad M = A_0 + A_1 * M,$$

and has a unique solution, since  $I - A_1$  is nonsingular. It follows, therefore, a unique steady state exists and the equilibrium sequence of  $M_t$  converges to it. Moreover,  $0 < S(A_1) < 1$  implies that  $\{M_t\}$  constitutes a monotone sequence<sup>7</sup>.  $\square$

PROOF OF PROPOSITION 2: In the thesis, we assume everyone is self-employed and each agent has her own office and equipments to produce outputs. They have the same production technology but different physical capital and human capital which distribute log-normally, i.e.,  $\ln k^i \sim N(m_k, \Delta_k^2)$  and  $\ln h^i \sim N(m_h, \Delta_h^2)$ . Then from the property of the moment generating function for lognormal distribution, we get

$$(A.54) \quad \ln k = m_k + \Delta_k^2/2 \text{ and } \ln h = m_h + \Delta_h^2/2,$$

where  $\Delta_h^2$  and  $\Delta_k^2$  are given by (39) and (38). Substituting (A.54) into (A.20) yields

$$(A.55) \quad y = k^\lambda h^\mu t^{1-\lambda-\mu} \exp \left( \left( (\lambda - 1) \lambda \Delta_k^2 + (\mu - 1) \mu \Delta_h^2 + 2\lambda\mu cov \right) / 2 \right) \\ = f(\kappa, \eta, \rho, \alpha, \beta, \lambda, \mu, \sigma^2, \tau) \\ \times \exp \left( \frac{\left( \begin{array}{c} \left( \begin{array}{c} (\lambda - 1)(1 - \alpha\lambda(1 - \tau)) \\ + 2\lambda\beta\mu(1 - \tau)^2 \end{array} \right) \lambda \Delta_k^2 \\ - \left( \begin{array}{c} 1 - \mu - \alpha\lambda(1 + \mu)(1 - \tau) \\ - 2\lambda\mu\beta(1 - \tau)^2 \end{array} \right) \mu \Delta_h^2 \end{array} \right)}{2(1 - \lambda(1 - \tau)(\alpha + 2\beta\mu(1 - \tau)))} \right),$$

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<sup>7</sup>For details, see page 255, Lorenz (1993).

where

$$(A.56) \quad \begin{aligned} k &= e^{m_k + \Delta_k^2/2} \\ &= f_1(\kappa, \eta, \rho, \alpha, \beta, \lambda, \mu, \varphi, \sigma^2, \tau), \end{aligned}$$

$$(A.57) \quad \begin{aligned} h &= e^{m_h + \Delta_h^2/2} \\ &= f_2(\kappa, \eta, \rho, \alpha, \beta, \lambda, \mu, \varphi, \sigma^2, \tau), \end{aligned}$$

$$(A.58) \quad m_k = \frac{1}{(1-\lambda)(1-\alpha) - \beta\mu} \left( \begin{aligned} &\mu(\ln \kappa + \varphi + \beta \ln \bar{s}_1) + (1-\alpha - \beta\mu) \ln \bar{s}_2 \\ &+ (1-\alpha)(1-\lambda - \mu) \ln l + (1-\alpha)\tau(2-\tau)\Lambda \end{aligned} \right),$$

$$(A.59) \quad m_h = \frac{1}{1-\alpha - \beta\mu} \left( \begin{aligned} &\ln \kappa + \varphi + \beta \ln \bar{s}_1 + \beta(1-\lambda - \mu) \ln l \\ &+ \beta\lambda m_k + \beta\tau(2-\tau)\Lambda \end{aligned} \right).$$

By (A.55), we can get (37). Then the Proposition 2 is proved.  $\square$

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